Uncertainty Analysis in QUAL2E Model of Zayandeh-Rood River

A. Abrishamchi, M. Tajrishi, P. Shafieian

ABSTRACT: Water-quality modeling and prediction is a complicated task because of inherent randomness and uncertainties associated with various processes and variables throughout the stream environment and the lack of appropriate data. Hence, the results of mathematical models are always approximate, lying within an uncertainty. This paper describes and demonstrates the application of the U.S. Environmental Protection Agency's water-quality model, QUAL2E-UNCAS, to the Zayandeh-Rood River in Iran. First-order reliability analysis is used to examine the variability of predicted water-quality parameters of total dissolved solids, dissolved oxygen, and biochemical oxygen demand. This analysis also determines key sources of uncertainty affecting prediction of the water-quality parameters. The results show that reliability analysis can help water-quality modelers and planners to quantify the reliability of the water-quality predictions and to carry out more efficiently planned sampling and data collection programs to reduce model-prediction uncertainty. *Water Environ. Res.,* **77,** 000 (2005).

KEYWORDS: streamwater quality, uncertainty analysis, first-order reliability analysis, QUAL2E model, Zayandeh-Rood River.

Introduction

Population growth and expansion of agricultural, industrial, and urban sectors are generally a threat to the integrity of water resources. Rivers in Iran, as fragile ecosystems and important sources of water for various purposes, are seriously affected by the above-mentioned activities in their basins. Hence, environmental concern has focused increasingly on the deterioration of the nation's rivers because of increased discharges of pollutants. This alarming situation requires effective measures to protect these vital ecosystems. Streamwater-quality protection and management needs a sufficient tool to describe the present situation and to predict changes expected from alternative management strategies. Mathematical modeling of the streamwater quality is a popular supporting tool in water-quality management.

Water-quality modeling and prediction is a complicated task, mainly because of inherent randomness exhibited throughout the stream environment. Not only are the physical and biological processes not clearly defined, but an imposing number of uncertainties are associated with the various processes occurring within the stream environment. The complex interaction and stochastic nature of water-quality variables prevent complete confidence in deterministic solution of a system's water quality.

Results of mathematical models are always approximate, lying within an uncertainty range. Application of streamwater-quality models on management process has been hampered by a lack of appropriate data for minimization of model simulation uncertainty. Uncertainty in simulation from water-quality models may contribute to the unexpectedly poor results of some stream water pollution control strategies and plans.

Water-quality data collection is a time-consuming and relatively expensive task. Thus, water-quality models developed and used for many rivers have been calibrated and verified with inadequate data collected during short periods of model development or before model development during general streamwater-quality monitoring. Under this situation, water quality is simulated with uncertain model parameters or inputs, thus increasing the uncertainty of simulation results and adversely affecting decision making for water pollution control.

This paper describes and demonstrates the application of the U.S. Environmental Protection Agency's (U.S. EPA's) water-quality model, QUAL2E-UNCAS (Brown and Barnwell, 1987), for the Zayandeh-Rood River in Isfahan, Iran. First-order reliability analysis (FORA), as one of the uncertainty analysis options provided by the UNCAS portion of this model, is used to examine the variability of predicted water-quality values. The FORA application provides insight on model performance in terms of key parameters and input variables requiring detailed study and the overall model prediction uncertainty. The objective of this study was to identify the model input parameters and variables (reaction coefficients, initial conditions, pollutant loads, etc.) that significantly affect the uncertainty of estimates of various key water-quality constituents so that the uncertainties in the coefficients can be reduced by a carefully designed sampling program.

Methodology

Modeling Uncertainty. The mathematical modeling process can be resolved into the transformation of input variables to output variables, using a given number of parameters and transfer function. A mathematical model has a certain mathematical form or structure and a set of parameters and can be represented by a relationship of the form (Freissinet et al., 1996). ?1

$$Y = F(X_i, a_j) + e \tag{1}$$

Where

Y = output variable,
X_i = input variable,
a_j = model parameters, and
e = conceptualization error.

Differences always exist between the true and estimated models and between the true and estimated model parameters. These differences represent modeling uncertainties. To prove that the model is not a rough approximation of reality, one has to make sure to choose adequate physical laws to describe the physical reality of the systems (cause-and-effect relationships) and to take the uncertainty in the model parameters and input variables into account.

Analysis of Uncertainty and Imprecision in Input Parameters. The problem of uncertainty in the results of mathematical modeling for ecologic systems is partly because of uncertainty of data and vague expert knowledge, which may result from incomplete or inadequate data, spatial and temporal variability in the parameters, estimation instead of measurements, and qualitative and subjective information from expert knowledge. Variability, or error in model inputs, contributes to uncertainty of calculated output variables. Analysis of the uncertainty of predicted values is necessary to determine the reliability with which the planner can use predicted water-quality values.

Different methods are available to estimate the validity and reliability of the results. Classical methods of sensitivity analysis and reliability analysis have been used for many years. Fuzzy rule-based approach is another approach which has found application in analysis of uncertainty and vagueness in hydrologic and ecologic parameters (Bardossy and Duckstein, 1995).

Classical Methods. Two well-known approaches exist for determining the effect of parameter uncertainty on the confidence of model results. The first one, called *sensitivity analysis*, consists essentially in perturbing each coefficient in a defined range of values, solving the equations, and observing the effect of the perturbation on the solution. A number of researchers (e.g., Beck, 1987; Gardner et al., 1981; Yeh and Tung, 1993) have shown that the traditional sensitivity analysis is not appropriate for determining the sources of uncertainty that most affect model output. The reason is that the sensitivity coefficient for a parameter does not account for the likelihood that the parameter is different from its "correct" value. Therefore, a highly sensitive parameter that is known with low uncertainty may have much less effect on the uncertainty of model output than a much less sensitive parameter that is highly uncertain.

The second approach is uncertainty analysis by which the uncertainty of the solution resulting from the uncertainty of one or more input parameters is estimated. The methods available for this kind of analysis can be classified into three main groups.

(1) Statistical sampling (or simulation-based) methods, which evaluate the range of likely output estimates by defining a representative set of values for the uncertain parameter as inputs for the model. The set of values is determined using random sampling methods, taking into account the probability distribution functions and correlation between the parameters. A well-known method belonging to this approach is Monte-Carlo simulation.
(2) First-order reliability analysis based on a Taylor series expansion and truncated after the first-order term (Benjamin and Cornell, 1970; Cornell, 1972). This requires knowledge of the sensitivity coefficient and covariance structure for each parameter.
(3) Bayesian methods, typically used when parameter values can only be specified by expert judgments, require many assumptions concerning the applications (Eslinger and Sagar, 1987).

Statistical sampling (e.g., Monte-Carlo Simulation) and I allow consideration of the combined effects of parameter sensitivity and parameter uncertainty in the determination of the key parameters affecting model prediction uncertainty.

First-Order Analysis of Uncertainty. Essentially, first-order analysis provides a methodology for obtaining an estimate for the moments of a random variable, which is a function of one or several random variables. It estimates the uncertainty in a mathematical model involving uncertain parameters. By using first-order analysis, the combined effect of uncertain parameters and the uncertainty in a model formulation can be assessed.

To present the methodology of first-order analysis, consider a random variable, C, as the concentration of the constituent simulated in the selected water-quality model, which is a function of N uncertain (i.e., random) variables (model-input variables, model parameters, etc.). Mathematically, C can be expressed as

$$\mathrm{C} = \mathrm{g}(\mathrm{X}) \tag{2}$$

Where $X = (x_1, x_2, \ldots, x_N)$; a vector containing N random variables x. In FOR A, a Taylor series expansion of the model output C is truncated after the first-order term

$$C = g(X_e) + \sum_{i=1}^{N}(x_i - x_{ie})\left[\frac{\partial g}{\partial x_i}\right]_{x_e} \tag{3}$$

Where X_e equals the vector of uncertain basic variables representing the expansion point and the subscript x_e indicates that the partial derivative is taken at the expansion point.

In FORA applications to water resources engineering, the expansion point is commonly the mean value of the basic variables. Thus, the expected value and variance of the performance function are as follows.

$$E[C] = \mu_c \approx g(X_m) \tag{4}$$

$$Var(C) = \sigma_c^2 \approx \sum_{i=1}^{N}\sum_{j=1}^{N}\left[\frac{\partial g}{\partial x_i}\right]_{X_m}\left[\frac{\partial g}{\partial x_j}\right]_{X_m}\mathrm{cov}[x_i, x_j] \tag{5}$$

Where

μ_c = mean of C;
σ_c = standard deviation of C;
X_m = vector of mean values of the basic variables; and
cov $[x_i, x_j]$ = covariance between random variables x_i and x_j.

If the basic variables are statistically independent (uncorrelated), eq 5 reduces to the following.

$$Var(C) = \sigma_c^2 \approx \sum_{i=1}^{N}\left[\frac{\partial g}{\partial x_i}\right]_{X_m}^2 \sigma_i^2 \tag{6}$$

Where $\sigma_{\text{for a}}$ = the standard deviation of basic variable i.

First-order analysis only provides estimates of the mean and variance of C; it does not give the probability distribution itself, which requires specification of all the moments (in the special case of a normal distribution, the first two moments completely specify the distribution). Despite this limitation, FOR A is an extremely useful tool in many applications. Its use is desirable in many cases where an approximate analysis is needed with only the means and variances of the random variables available, a situation often found in practical cases. The use of first-order analysis is justified particularly when the estimates for higher moments of a distribution are very uncertain because of small sample size.

It is typically assumed that C is normally distributed, and the exceedance probability P_E for a given target concentration C_T is estimated as

$$P_E = 1 - \phi\{(C_T - \mu_c)/\sigma_c\} \tag{7}$$

Where $\phi\{\,\}$ is the standard normal integral. The normal assumption has several practical advantages that are discussed by Yen et al. (1986) and Melching (1995).

When the Taylor series is expanded at the mean of the basic variables, only mean and variance of the basic variables and simple sensitivity coefficients are required in FORA. However, its application to engineering design problems has several theoretical or conceptual problems. The main problem is that a single linearization of the model output function at the central value of the basic variables is assumed to represent the statistical properties of model output over the complete range of basic variables values. For nonlinear systems, this assumption becomes more inaccurate because design failure should result only because of extreme values of the basic variable describing the system (Cheng, 1982; Melching, 1992).

First-order reliability analysis has been widely used in all fields of engineering because of its relative ease in application to many types of problems. The FORA, with the expansion at the mean values, has been applied successfully in the water-quality modeling, despite the conceptual problems. Burges and Lettenmaier (1975) and Melching and Anmangandla (1992) have applied the method to investigate the uncertainty in prediction of biochemical oxygen demand (BOD) and dissolved oxygen (DO) within a stochastic stream environment. Chadderton et al. (1982) used FORA to determine the relative contributions of reaeration rate, deoxygenation rate, initial DO concentration, and BOD load on output uncertainty for the Streeter-Phelps model (Streeter and Phelps, 1925) for streamflow conditions typical for natural streams. Brown and Barnwell (1987) used FORA to determine the relative contributions of all parameters in QUAL2E on the uncertainty of estimates of carbonaceous BOD (CBOD) and DO concentrations for a river in Georgia and Florida. Melching and Yoon (1996) illustrated that a simple method based on FORA may be applied to determine key sources of uncertainty affecting uncertainty for complex water-quality models.

QUAL2E-UNCAS Stream-Quality Model. Water-quality models have been used since 1925, when Streeter and Phelps (1925) developed the first water-quality model. As a steady-state model, QUAL2E is presently the most widely used model for simulating streamwater quality. It is capable of simulating up to 15 water-quality constituents in branched stream networks that are well-mixed laterally and vertically. Among its many capabilities, it allows for multiple waste discharges, withdrawals, tributary flows, and incremental (that is, distributed) flows and outflows.

Constituents that can be simulated in the model are DO, BOD, temperature, algae as chlorophyll a, components of the nitrogen cycle as nitrogen (organic nitrogen, ammonia, nitrite, and nitrate), components of the phosphorus cycle (like organic and dissolved phosphorus), coliforms, an arbitrary nonconservative constituent, and three arbitrary conservative constituents. The primary application of QUAL2E is simulation of DO and CBOD, the nitrogen cycle, algae (dependent on the nitrogen and phosphorus cycle), sediment oxygen demand, and atmospheric reaeration. Details on these interactions as simulated in QUAL2E are presented in Brown and Barnwell (1987).

In QUAL2E model simulations, the stream is conceptualized as a string of completely mixed reactors that are linked sequentially by advective transport and dispersion. Sequential groups of these reactors or computational elements are defined as *reaches.* Computational elements in each reach have identical length, hydrogeometric properties, and biological rate constants. The hydrogeometric properties and biological rate constants may change between reaches, but the computational element length remains constant throughout the simulated stream.

A mass balance is used to keep track of the water quality constituents. This balance can be written generally as

$$\underbrace{V\frac{\partial c}{\partial t}}_{Accumulation} = \underbrace{\underbrace{\frac{\partial\left(A_c E\frac{\partial c}{\partial x}\right)}{\partial x}dx}_{Dispersion} - \underbrace{\frac{\partial(A_c U c)}{\partial x}dx}_{Advection}} + \underbrace{V\frac{dc}{dt}}_{Kinetics} + \underbrace{s}_{External\ sources/sinks} \quad (8)$$

Where

V = volume,
C = constituent concentration,
A_c = element cross-sectional area,
E = longitudinal dispersion coefficient,
x = distance,
u = average velocity, and
s = external sources (positive) or sinks (negative) of the constituents.

The model solves the finite difference formulation of the above one-dimensional advection dispersion equations applied to successive stream reaches.

The QUAL2E-UNCAS is a version of the QUAL2E model that allows consideration of parameter uncertainty in predicting water-quality variables. Uncertainty subroutines are included in QUAL2E (QUAL2E-UNCAS) with options for sensitivity analysis, FORA, and Monte Carlo simulation. In this paper, FORA option of QUAL2E-UNCAS is applied to determine the relative contributions of some important parameters in QUAL2E on the uncertainty of estimates of BOD and DO concentrations for the Zayandeh-Rood River in Isfahan.

Case Study

Description of the Zayandeh-Rood River. The Zayandeh-Rood (ZR) River is the main perennial river in central Iran with a length of approximately 320 km. The total catchment area of this river is approximately 31 000 km^2, of which 4200 km^2 is upstream of the ZR Reservoir, located 110 km northwest of the city of Isfahan. The ZR River flows toward the Southeast and drains in to Batlaq-e-Gavkhooni (Gavkhooni marsh), 130 km southeast of the city of Isfahan. The river's flow, plus the augmentation by transbasin diversion from Upper Karoon River tributaries (Koohrang River), constitutes the surface water resources for urban, agricultural, and industrial uses in the Isfahan plain. It provides water for several cities, including Isfahan, with a total population of over two million and for many large industries located in the river basin. It also irrigates over 100 000 hectares of land before leaving the Isfahan plain. The watershed of the ZR River and the modeled portion of the river are shown in Figure 1.

The ZR river basin may be divided into three different zones based mainly on the topography and climate.

(1) The high mountainous areas at the western boundary of the basin are the main part of the watershed which feed the ZR River, with an annual average precipitation of approximately 1000 mm, mainly in the form of snow.
(2) The lower mountainous areas have a lesser amount of precipitation (200 to 300 mm per year).
(3) The plain area, which includes the major agricultural, industrial, and urban zones, has a low amount of precipitation (approximately 100 mm per year).

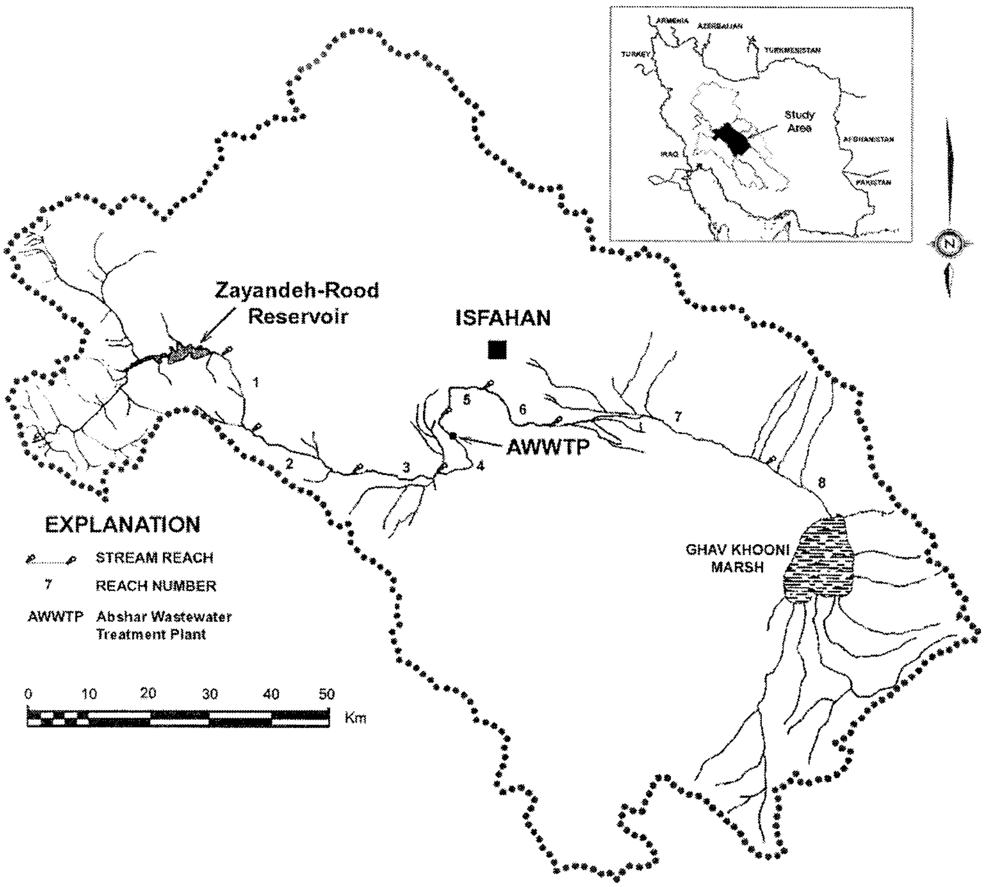

Figure 1—Location of Zayandeh-Rood River Basin and model reaches simulated in QUAL2E – Zayandeh-Rood.

The water quality of the ZR River in the lower zone (i.e., plain area) is influenced by municipal, industrial, and agricultural pollution. There are several cities and villages along the river which discharge their raw or partially treated wastewater with an average five-day BOD (BOD_5) of over 200 mg/L to the river. The Isfahan wastewater treatment plant (WWTP) (Abshar Plant), as the biggest point source, discharges its effluent directly to the river (see Figure 1). This plant is hydraulically overloaded frequently during the year; thus raw wastewater bypasses the plant and discharges directly to the river.

Several large industries (mainly textile) also discharge their raw or partially treated wastewater into the river. Another major source of ZR River pollution is agricultural activities. Drainage water in the form of point source (open drains) and distributed source (interflow) is discharged to the river. Urban runoff is another source of ZR River pollution.

Application of the QUAL2E-UNCAS Model. Among several models, the QUAL2E model was selected to use in the study because the flow condition downstream of ZR Reservoir is relatively steady. The modeled portion of the river was divided into eight reaches containing conceptual elements that were each 500 m long (Figure 1).

Model Calibration. Calibration is an important step in the modeling procedure, especially when the model is used for prediction purposes. Model calibration aims at determining the values for a subset of parameters which can not be obtained directly from measurement.

The QUAL2E model was calibrated for application to the ZR River by adjusting their parameters to ensure the ability to simulate historic data. Data collected in February 1997 were used for model calibration (Haghighi et al., 1997). In the data collection survey, multiple samples taken from the river were analyzed for several constituents and quality parameters including BOD_5, DO, and total dissolved solids (TDS).

The prediction capacities of QUAL2E for DO, BOD, and TDS were investigated by adjusting the reaeration rate and deoxygenation rate coefficients within the expected range presented by Brown and Barnwell (1987). In reality, it is not possible to get full agreement between modeled and observed values because the input data typically represent average annual and monthly values, and therefore model results will also describe the mean water-quality conditions while observed values represent only instant events.

To verify the model, the summer month of June 1988 data and the values of the coefficients obtained from calibration were used. During verification, best predictions were obtained for DO concentration.

Determination of Key Sources of Uncertainty. The FORA was applied to the calibrated model to determine the model parameters and input variables that significantly contributed to uncertainty in QUAL2E estimating of DO, BOD, and TDS concentrations at the selected locations in the river. The uncertainty of each parameter was selected at the default values for typical QUAL2E application in Brown and Barnwell (1987) and are listed in Table 1.

Table 1—Estimated coefficient of variation values for input parameters and variables.

Parameter (variable) definition	Coefficient of variation
CBOD decay rate	15%
Reaeration rate coefficient	15%
Initial temperature	2%
Headwater flowrate	5%
Headwater oxygen concentration	5%
Headwater BOD	10%
Headwater TDS	3%
Point discharge rate	5%
Point discharge oxygen concentration	5%
Point discharge BOD	10%
Point discharge TDS	3%

Derivatives required in FORA were determined numerically by increasing the parameter values one at a time by 5%, determining the change in concentration of the constituent of interest, and dividing the change in concentration by the increase in the parameter value to obtain the normalized sensitivity coefficient. The application of a 5% increment in the parameter values was recommended by Brown and Barnwell (1987) for uncertainty calculations in QUAL2E-UNCAS. The normalized sensitivity coefficients, $SN_{j,i}$, are mathematically defined as

$$SN_{j,i} = (\Delta C_j / C_{jo})(\Delta X_i / X_{io}) \tag{9}$$

Where

ΔC_j = change in the estimated concentration of constituent j resulting from change ΔX_i in parameter i with all other parameters kept at their original values;
C_{jo} = estimated concentration of constituent j when all parameters are at their original values; and
X_{io} = original value for parameter i.

The locations selected to study model-prediction uncertainty analysis were at reach 2 element 8, reach 5 element 7, and reach 6 element 19 at the river kilometers of 70, 160, and 260, respectively. These locations are fairly evenly distributed throughout the river with diverse values of the model parameters.

The calculated normalized sensitivity coefficients of TDS, DO, and BOD are listed in Tables 2, 3, and 4, respectively. The contribution of key model parameters to the variance in the TDS, DO, and BOD concentrations at the selected locations are listed in Tables 5, 6, and 7, respectively.

Table 2—Normalized sensitivity coefficients of total dissolved solids to key model input parameters.

Input parameter (variable)	Normalized sensitivity coefficients		
	Reach 2 element 8	Reach 5 element 7	Reach 6 element 19
Headwater flowrate	−0.364	−0.418	−2.315
Headwater TDS	0.172	0.081	0.004
Point discharge rate	0.372	0.424	2.915
Point discharge TDS	0.828	0.919	0.996

Note: The remaining parameters have normalized sensitivity coefficients <0.1.

Table 3—Normalized sensitivity coefficients of dissolved oxygen to key model input parameters.

Input parameter (variable)	Normalized sensitivity coefficients		
	Reach 2 element 8	Reach 5 element 7	Reach 6 element 19
BOD decay rate	0	−0.455	−0.342
Reaeration rate coefficient	0.095	0.539	0.555
Temperature	−0.175	−0.935	−0.792
Headwater flowrate	0.575	0.930	0.836
Headwater DO	0.167	0.079	0.048
Point discharge	−0.639	−0.991	−0.943
Point discharge DO	0.413	0.333	0.350
Point discharge BOD	0	−0.687	−0.629

Note: The remaining parameters have normalized sensitivity coefficients <0.1.

To carry out sensitivity analysis, the key parameters are identified by ranking the normalized sensitivity coefficients. The rankings of the most important parameters affecting the estimates of TDS, DO, and BOD obtained from normalized sensitivity analysis and FORA are presented in Tables 8, 9, and 10, respectively.

As Table 8 shows, TDS estimation is most sensitive to TDS point loads, headwater flowrate, and point-discharge rate. The effect of these parameters increases along the river. The concentration of headwater TDS has less effect on TDS concentration estimates and its effect decreases in the downstream direction. These results are expected as the river TDS concentration increases along the river because of water withdrawals, consumption, and discharges of drain water along the river. However, the sensitivity of TDS concentration estimates to headwater flow rate increases in downstream reaches because of the diluting role of headwater flow. The ranking of key parameters applying normalized sensitivity analysis and FORA are the same.

The rankings of key parameters affecting estimated DO concentration applying normalized sensitivity analysis and FORA (Table 9) differ substantially and change along the river. As is expected, the contribution of reaeration rate, point-discharge BOD, and BOD decay rate to the variance in estimate of DO concentration

Table 4—Normalized sensitivity coefficients of biochemical oxygen demand to key model input parameters.

Input parameter (variable)	Normalized sensitivity coefficients		
	Reach 2 element 8	Reach 5 element 7	Reach 6 element 19
BOD decay rate	−0.027	−0.591	−0.253
Temperature	−0.025	−0.658	−0.289
Headwater flowrate	−0.251	−0.595	−1.772
Headwater BOD	0.163	0	0
Point discharge	0.239	0.784	2.268
Point discharge BOD	0.837	1	1

Note: The remaining parameters have normalized sensitivity coefficients <0.1.

Table 5—Contribution of key model input parameters to total dissolved solids concentration variance.

	Reach 2 element 8			Reach 5 element 7			Reach 6 element 19		
Input parameter	VAR[a]	VAR[a] %	CV[b]	VAR[a]	VAR[a] %	CV[b]	VAR[a]	VAR[a] %	CV[b]
Headwater flowrate	59.1	25	0.018	172.6	26.5	0.020	1652666	37.7	c
Headwater TDS	4.8	2	0.005	2.4	0.4	0.002	2	0	c
Point discharge rate	62	26.2	0.019	177	27.1	0.021	2621161	59.8	c
Point discharge TDS	110.3	46.7	0.025	299.5	46	0.028	110131	2.5	c
Sum	236.2	100	0.326(1)	651.5	100	0.041	4383960	100	c
Mean TDS		423			628			—[c]	

[a] VAR = Variance.
[b] CV = Coefficient of variation of TDS concentration prediction.
[c] Model can not simulate TDS concentration over 10 000 mg/L.
Note: The remaining input parameters contribute <1% to the variance.

increases along the river and are the most affecting parameters at middle reaches of the river where the effluent of the Abshar WWTP is discharged to the river.

Regarding the estimate of BOD concentration, point-discharge BOD, headwater flowrate, point discharges, headwater BOD, BOD decay rate, and temperature have the highest normalized sensitivity coefficients at the upper reaches of the river. The ranking of these coefficients changes along the river. The point discharge BOD, BOD decay rate, point discharge, and headwater flowrate are the key model parameters most contributing to the variance in the highest BOD concentration at the middle reaches of the river where the Abshar WWTP.

Constituent Prediction Uncertainty Along the River. The mean, variance, and coefficient of variation of TDS, DO, and BOD concentration estimates at three points along the river determined by FORA are listed in Tables 5, 6, and 7, respectively. The statistics of standard deviation (or variance) is used for comparing the relative contribution of uncertainty (i.e., ranking) of different parameters in uncertainty of the estimates of constituent concentrations. To compare the degree of uncertainty of different constituents, one has to compare their coefficient of variation (defined as the standard deviation divided by the mean) of different constituent estimates. As the middle and downstream locations of the river are more critical in respect to pollution and need of control, the constituent prediction uncertainty is discussed at these locations.

The TDS estimates have a very small coefficient of variation, indicating that model parameter uncertainty has insignificant effects on prediction uncertainty for this constituent in the QUAL2E ZR River estimation. The coefficient of variation of DO and BOD concentration is fairly high. Thus, the parameters significantly affecting the prediction uncertainty of DO and BOD, as indicated in Tables 9 and 10, require additional sampling and more accurate measurement to reduce parameter uncertainty.

Summary and Conclusions

This paper has illustrated the applicability of FORA, a simple reliability analysis method, in determining key sources of uncertainty affecting prediction uncertainty for Zayandeh-Rood River QUAL2E model. The FORA was applied to determine the key parameters affecting prediction uncertainty for TDS, DO, and BOD along the Zayandeh-Rood River, simulated with QUAL2E. The reliability analysis considered uncertainties in 11 model parameters. The uncertainty of estimated concentrations of TDS because of input-variable uncertainty was found to be acceptably small. Therefore, data collection to refine variables significantly affecting this constituent would not greatly reduce model prediction

Table 6—Contribution of key model input parameters to dissolved oxygen concentration variance.

	Reach 2 element 8			Reach 5 element 7			Reach 6 element 19		
Input parameter	VAR[a]	VAR[a] %	CV[b]	VAR[a]	VAR[a] %	CV[b]	VAR[a]	VAR[a] %	CV[b]
BOD decay rate	0	0	0	0.13	22	0.07	0.08	14.6	0.05
Reaeration rate coefficient	0.01	8	0.01	0.18	30.8	0.08	0.22	38.4	0.08
Temperature	0.0006	0.48	0.003	0.0097	1.65	0.02	0.008	1.39	0.02
Headwater flowrate	0.043	32.5	0.03	0.06	10.23	0.05	0.056	9.67	0.04
Headwater DO	0.004	2.73	0.008	0.0004	0.07	0.004	0.0002	0.03	0.002
Point discharge rate	0.053	39.8	0.03	0.068	11.6	0.05	0.072	12.31	0.05
Point discharge DO	0.022	16.7	0.02	0.008	1.31	0.02	0.01	1.70	0.02
Point discharge BOD	0	0	0	0.131	22.3	0.07	0.128	21.9	0.06
Sum	0.132	100	0.05(1)	0.59	100	0.15	0.58	100	0.14
Mean DO		7.2			5.3			5.7	

[a] VAR = Variance.
[b] CV = Coefficient of variation of DO concentration prediction.
Note: The remaining input parameters contribute <1% to the variance.

Table 7—Contribution of key model input parameters to biochemical oxygen demand concentration variance.

	Reach 2 element 8			Reach 5 element 7			Reach 6 element 19		
Input parameter	VAR[a]	VAR[a] %	CV[b]	VAR[a]	VAR[a] %	CV[b]	VAR[a]	VAR[a] %	CV[b]
BOD decay rate	0	0.2	0	0.195	38.5	0.09	0.213	4.5	0.04
Headwater flowrate	0	2	0	0.02	4.3	0.03	1.16	24.4	0.09
Headwater BOD	0	3.5	0	0	0	0	0	0	0
Point discharge rate	0	1.9	0	0.04	7.5	0.04	1.9	40	0.11
Point discharge BOD	0	92.3	1	0.25	48.8	0.1	1.48	31	0.1
Sum	0	100	2[(1)]	0.51	99	0.15	4.7	100	0.18
Mean BOD		0.02			5			12.2	

[a] VAR = Variance.
[b] CV = Coefficient of variation of BOD concentration prediction.
Note: The remaining input parameters contribute <1% to the variance.

uncertainty. The uncertainty of estimated concentrations of DO and BOD resulting from model parameter or input variable uncertainty was found to be significant. However, model parameters such as reaeration-rate and BOD decay-rate coefficient and the model input variables such as the point discharge, point-discharge BOD, and headwater flowrate have a dominant effect on prediction uncertainty. Reduction of the uncertainty in these parameters and variables could significantly improve model prediction uncertainty of DO and BOD. These results show that reliability analysis can help water quality modelers and planners to quantify the reliability of the water quality predictions and to carry out more efficiently planned sampling and data collection programs to reduce model prediction uncertainty.

Last, but not least, it should be emphasized that model prediction uncertainty owing to parameter and variable uncertainty alone was analyzed and no model uncertainty was taken into account.

Table 8—Comparison of ranking of key parameters affecting estimated total dissolved solids concentrations applying normalized sensitivity analysis and first-order reliability analysis.

		Rank among four parameters estimated by	
Reach-element	Model parameter	Normalized sensitivity analysis	FORA
2–8	Headwater flowrate	3	3
	Headwater TDS	4	4
	Point discharge rate	2	2
	Point discharge TDS	1	1
5–7	Headwater flowrate	3	3
	Headwater TDS	4	4
	Point discharge rate	2	2
	Point discharge TDS	1	1
6–19	Headwater flowrate	2	2
	Headwater TDS	4	4
	Point discharge rate	1	1
	Point discharge TDS	3	3

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Table 9—Comparison of ranking of key parameters affecting estimated dissolved oxygen concentrations applying normalized sensitivity analysis and first-order reliability analysis.

		Rank among eight parameters estimated by	
Reach-element	Model parameter	Normalized sensitivity analysis	FORA
2–8	BOD decay rate	7	8
	Reaeration rate coefficient	6	4
	Temperature	4	6
	Headwater flowrate	2	2
	Headwater DO	5	5
	Point discharge	1	1
	Point discharge DO	3	3
	Point discharge BOD	8	7
5–7	BOD decay rate	6	3
	Reaeration rate coefficient	5	1
	Temperature	2	6
	Headwater flowrate	3	5
	Headwater DO	8	8
	Point discharge	1	4
	Point discharge DO	7	7
	Point discharge BOD	4	2
6–19	BOD decay rate	7	3
	Reaeration rate coefficient	5	1
	Temperature	3	7
	Headwater flowrate	2	5
	Headwater DO	8	8
	Point discharge	1	4
	Point discharge DO	6	6
	Point discharge BOD	4	2

Table 10—Comparison of ranking of key parameters affecting estimated biochemical oxygen demand concentrations applying normalized sensitivity analysis and first-order reliability analysis.

Reach-element	Model parameter	Rank among six parameters estimated by: Normalized sensitivity analysis	FORA
2–8	BOD decay rate	5	5
	Temperature	6	—
	Headwater flowrate	2	3
	Headwater BOD	4	2
	Point discharge	3	4
	Point discharge BOD	1	1
5–7	BOD decay rate	5	2
	Temperature	3	—
	Headwater flowrate	4	4
	Headwater BOD	6	5
	Point discharge	2	3
	Point discharge BOD	1	1
6–19	BOD decay rate	5	4
	Temperature	4	—
	Headwater flowrate	2	3
	Headwater BOD	6	5
	Point discharge	1	1
	Point discharge BOD	3	2

?2 **Authors.** A. Abrishamchi is an Associate Professor at the Department of Civil Engineering, Sharif University of Technology, Iran, and a member of the Iranian Water Resources Association (IRWRA). M. Tajrishi is an Assistant Professor, and P. Shafieian is a graduate student at the Department of Civil Engineering, Sharif University of Technology.

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