Using satellite data to extract volume–area–elevation relationships for Urmia Lake, Iran

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A B S T R A C T

Urmia Lake in the northwest of Iran is the second largest hyper-saline lake worldwide. During the past two decades, a significant water level decline has occurred in the lake. The existing estimations for the lake water balance are widely variable because the lake bathymetry is unknown. The main focus of this study is to extract the volume–area–elevation (V–A–L) characteristics of Urmia Lake utilizing remote sensing data and analytical models. V–A–L equations of the lake were determined using radar altimetry data and their concurrent satellite-derived surface data. Next, two approximate models, a power model (PM) and a truncated pyramid model (TPM), were parameterized for Urmia Lake and checked for accuracy. Results revealed that in comparison with the satellite-derived reference volume–elevation equation, the PM slightly over-predicts the volume of Urmia Lake while the TPM under-estimates the lake storage. Variations of the lake area and volume between 1965 and 2011 were examined using the developed V–A–L equations. Results indicated that the lake area and volume have declined from the historical maximum values by 2200 km² and 33 km³, respectively. To restore Urmia Lake to a level to maintain ecological benefits, 13.2 km³ of water is required. This study demonstrates the use of remote sensing data of different types to derive V–A–L equations of lakes. Substituting satellite-derived V–A–L equations for common empirical formulas leads to more accurate estimations of a lake water balance, which in turn, provides insight to water managers for properly assessing and allocating water resources to downstream ecosystems.

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Introduction

Urmia Lake is the second largest hyper-saline lake worldwide. It is located in a closed basin between 37° 04′ N and 38° 17′ N latitude and 45° E and 46° E longitude in the northwest of Iran. It has an historical maximum surface area of 5700 km² (Alipour, 2006). Urmia Lake was declared a wetland of international importance by the Ramsar Convention in 1971 (Ramsar Convention website). Moreover, because of its ecological importance, the lake is defined as a National Park and International Biosphere Reserve (Abbaspour and Nazaridoust, 2007). Urmia Lake is situated in a semi-arid area, having a mean annual temperature of 11.2 °C, an average precipitation and evaporation rate of 341 and 1200 mm/yr, respectively (Djamali et al., 2008). Aquatic biodiversity is limited by the lake's high salinity. The most significant aquatic biota in the lake is a brine shrimp species, Artemia urmiana and no flora other than phytoplankton is found within the lake (Ghaheri et al., 1999). There are four major islands in the south part of the lake which are considered protected areas by the Iran Department of Environment (Fig. 1). These islands are important destinations for various migratory birds including flamingos, pelicans, spoonbills, ibises, storks, avocets, stilts and gulls. There are also two very rare species of mammals which are sheltered and preserved on the islands of Urmia Lake: Yellow Persian deer (Dama mesopotamica) and a variety of sheep (Ovis Orientalis gimelini).

During the last two decades the lake water level has significantly dropped mainly due to over-exploitation of upstream rivers and ongoing drought (Elmanfar and Mohabbi, 2007; Hassanzadeh et al., 2011). Retreat of Urmia Lake from its original shoreline is not only a hydrological concern, but it also presents serious challenges for water quality, conservation, human health and economics. For example, the decrease in volume has caused the salinity level of the lake to exceed the tolerable salinity threshold of A. urmiana. Population growth rates of A. urmiana are expected to decline as most of the species physiological activities will cease due to the increased salinity (Agh et al., 2008; Asem and Rastegar-Pouyani, 2010). Another consequence of the lake desiccation is the expansion of its islands, which has resulted in land bridges between some of the islands and the east shores. This has caused that some species of Persian Fallow Deer to flee from the islands to nearby villages. Moreover, precipitation of dissolved salts has produced salt crusts covering the black organic mud of the lake bed, particularly at the shorelines (Alipour, 2006). These dried coastal salt lands

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can create salt dusts that then are dispersed over the surrounding agricultural and residential areas when they are exposed to strong winds. It has been estimated that the 76 million people living within a 500 km radius of the lake will be at risk of such windblown salt-storms (UNEP and GEAS, 2012).

To overcome such challenges, it is essential to setup a comprehensive integrated water management plan which takes into account all elements of the basin’s water budget and can balance demands for human use and ecosystem requirements. The integrated management plan of Urmia Lake was established in May 2010 under the United Nations Development Program/-Global Environmental Facility/-Iran Department of Environment joint project for conservation of Iranian Wetlands (Department of Environment, 2010). Within this framework, it is noted that a volume of 3.1 km^3 of water needs to enter the lake annually in order to keep its water level at the minimum ecological level of 1274.1 m. This level was defined to meet the hydrological and water quality conditions (NaCl< 240 g/L) required to preserve A. urmiana, as the main ecological feature of Urmia Lake (Abbaspour and Nazaridoust, 2007).

One of the principal shortcomings of that study, which is considered as the base point for the lake restoration, is lack of sufficient knowledge about the lake bathymetry. Accuracy of the volume–area–elevation (V–A–L) and salt balance equations applied in that study is highly questionable because of large uncertainties in the few data used to derive them. A comprehensive bathymetric survey has never been conducted in Urmia Lake. Because the lake is shallow and hyper-saline, movement of ships and boats is prohibited. Thus, eco-sounder measurements cannot be applied over the whole lake. A few hand soundings have been made by local researchers, but these are insufficient to generate a Digital Elevation Map (DEM) of Urmia Lake. Analytical models expressing the volume–elevation relationship of water bodies from minimal field data can serve as geometric depression models in simulation studies (Nilsson et al., 2008). Application of analytical models in hydrologic studies to predict wetland volume characteristics has been considered for many years (Singh and Woolhiser, 2002). A number of researchers have developed different models for various types of water bodies. These models can be categorized into two major types: Power Models (PM) and Truncated Pyramid Models (TPM). O’Connor (1989) parameterized PMs for several lakes and reservoirs in the United States to simulate variation of dissolved solids. Wise et al. (2000) developed a volume–elevation (V–L) PM for an isolated marsh wetland. Furthermore, Hayashi and van der Kamp (2000) introduced a PM to represent the area–depth relations of ephemeral ponds and wetlands in small natural depressions. A PM was developed by Nilsson et al. (2008) to describe volume–elevation (V–L) relationships for different types of wetlands in the United States and Canada. Likewise, the TPMs have been applied by limnologists and fisheries biologists to compute volumes of lakes and wetlands (Taube, 2000). For example, Shjello (1968) used a truncated pyramid formula to verify that wetland volumes developed from specific topographic maps were accurate. Moreover, the TPM is currently used by several global lake databases such as the HYDROWEB (http://www.LEGOS.obs-ip.fr/soa/hydrologie/HYDROWEB) to calculate volume variations for a number of lakes.

Analytical models require some data from the lake V–A–L to be developed or validated, although not as intensive as bathymetric studies. Remote sensing appears to be an ideal method to acquire data for analytical models. Several researchers have confirmed the potential of remote sensing data to extract detailed information of wetlands such as wetland size, shape, type and extent (e.g. Cavalli
et al., 2009; Jollineau and Howarth, 2002). For instance, high resolution satellite images in conjunction with radar altimetry data can be effectively used to provide the area and water level data of lakes (e.g. DeVogel et al., 2004; Gao, 2009; Gao et al., 2012).

This study investigates the application of radar altimetry data in combination with high-resolution satellite images to extract the V–A–L characteristics of Urmia Lake. To accomplish this objective, first, altimetry data of the lake level was acquired and validated against gauge data. After exploiting the concurrent area of the lake from satellite imagery and the lake base level as the only required field data, the area–elevation relation (A–L) was developed and assessed for accuracy. Then, the V–L equation of Urmia Lake was derived by integrating the (A–L) equation and was applied as a reference formula to evaluate the approximate equations. Subsequently, two common analytical V–L models, a power model and a truncated pyramid model were parameterized to approximate the geometry of Urmia Lake. Finally, based on the extracted V–A–L equations, long-term variation of the lake surface area and volume was examined.

Methods

Data requirement and acquisition

Altimetry data

Satellite altimetry is a technique which can be applied in hydrological studies of water bodies such as water level monitoring. Originally, water level measurement by satellite altimetry was developed and optimized for open oceans. Subsequently, the technique has been used to study inland waters, particularly to remotely detect water surface level changes in lakes, inland seas, rivers, floodplains and wetlands (Aladin et al., 2005; Birkett, 1995; Crétaux and Birkett, 2006; Crétaux et al., 2005).

The lake global altimetry database is comprised of the merged Topex/Poseidon, Jason-1 and 2, ERS, ENVISAT and GFO data provided by ESA, NASA and CNES data centers (PODAAC and AVISO). Radar altimeters principally differ from satellite imaging devices because they just repeatedly retrieve the surface heights along a narrow swath determined by the instrument’s footprint size. Measurements of the water level are performed within a terrestrial reference frame at an interval from 10 to 35 days, depending on the orbit cycle of the satellite. The radar altimetry technique has been appropriately validated and applied as a robust measurement system. Further details on the methodology of producing satellite altimetry data can be found in Crétaux and Birkett (2006) and Crétaux et al. (2011).

Water level records of numerous large rivers, lakes and wetlands worldwide, can be obtained from the HYDROWEB data center (http://www.LEGOS.obs-ip.fr/soa/hydrologie/HYDROWEB). The database has been developed by the LEGOS (Laboratoire d’Etude en Géophysique et Océanographie Spatiale) in Toulouse, in coordination with the HYDROLARE project (Headed by SHI: State Hydrological Institute of the Russian Academy of Science). The HYDROWEB freely provides monthly water level data of nearly 160 large lakes and reservoirs extracted from multi-satellite altimetry measurements (Crétaux et al., 2011). Altimetry data of Urmia Lake from 1992 to 2010 were acquired from the HYDROWEB. These data were integrated from multiple satellites including Jason1 and 2, Topex/Poseidon, ENVISAT and GFO. Location of satellite tracks over Urmia Lake can be found from HYDROWEB.

Gauge data

Since 1965, water levels of Urmia Lake have been continuously measured against the chart datum (CD) in the Persian Gulf using a hydrostatic recording gauge at Golmankhaneh station (St.1 in Fig. 1). Due to the falling lake level, the location of the gauge was changed to a deeper place near the causeway in August 2008 (St.2 in Fig. 1). Having accuracy of some millimeters, these records fulfill the required accuracy of three millimeters at continuous-record gauging stations (WMO, 1994). Moreover, according to Vuglinskiy (2009) for the purpose of water balance studies, a daily measurement of level is usually sufficient in large lakes. Therefore, the daily water level records of Urmia Lake can be confidently used to validate the corresponding satellite-derived level data.

Lake area data

To assess the time series of Urmia Lake surface area, the HYDROWEB data center was used. The HYDROWEB lake area data have been provided through employing several high-resolution multispectral satellite imagers (e.g. Landsat-TM/ETM+) at different times. Using the corresponding satellite altimetry data, a rating curve function (dL/da), which represents the variation of water level with respect to the variation of surface area in a given time span, was calculated. Afterward, applying this rating curve function to the near continuous level data acquired from satellite radar altimetry, the surface areas of the lakes in the database over the time span of altimetry data (ranging from 1 to 2 days for big lakes) were estimated. Based on this principle, variation of the water level and surface for nearly 20 large lakes and reservoirs including Urmia Lake have already been calculated and provided by HYDROWEB (Crétaux et al., 2011).

For Urmia Lake, the rating curve function has been established using the extracted surface areas from 21 satellite images acquired between 1998 and 2010 and their corresponding altimetry data. Then, the areal extent of the lake for the period of altimetry data (1992 and 2010) was calculated with this formula.

Methodology of extracting the characteristic curves of Urmia Lake

The HYDROWEB water level data from Urmia Lake collected between 1992 and 2010 were acquired and compared to the in-situ data. Next, Urmia Lake rating curve function was modified according to the validated water level data. Subsequently, exploiting the modified rating curve, a time series of Urmia Lake surface area in the period from 1992 to 2010 was derived. Simultaneous validated time series of the water level and surface area allowed us to derive the A–L relation for Urmia Lake. Then, accuracy of the proposed equation was assessed using the lake surface area data obtained from several independent studies. The necessary steps for deriving the reference V–A–L equations are schematically presented in Fig. 2. Validation of the altimetry data can be considered as an arbitrary step (depicted with dash lines in Fig. 2), which should only be performed when gauge data are available. Otherwise, the procedure can be followed, although the uncertainty of the developed equation will be undetermined.

To estimate the lake V–L relation, three methods were employed. The first one is the integration of the A–L equation, while the two latter are the approximate analytical volume-depth models including a power-function model (PM) and a truncated pyramid model (TPM). The approximate models are parameterized for Urmia Lake and their performances were analyzed. Finally, long-term variation of the lake elevation, area and volume was examined based on the extracted characteristic curves.

Measures to assess accuracy of the approximate V–L equations

Accuracy of the approximate equations is assessed based on the two performance measures introduced by Nilsson et al. (2008): the normalized RMSE (RMSErel), and the absolute relative volumetric error (VARE). These two statistics are defined as follows:

\[
\text{RMSE}_{\text{rel}} = \frac{100}{V_{\text{max}}} \sqrt{\frac{1}{k} \sum_{i=1}^{k} (V_{\text{ref}} - V_{\text{M}})^2}
\]

\[
V_{\text{ARE}} = \frac{1}{k} \sum_{i=1}^{k} \left( \frac{V_{\text{M}} - V_{\text{ref}}}{V_{\text{ref}}} \right) \times 100\%
\]
where \( i \) = index number for lake state; \( k \) = total number of lake levels; \( V_{r} \) = lake volume derived from the reference equation at lake state \( i \); and \( V_m \) = volume produced from an approximate model (either from the power model \( V_P \) or from the truncated pyramid model \( V_T \)) at lake state \( i \). Furthermore, to determine whether the approximate models under or over-predicted the reference values of the lake volume, the relative volumetric error (\( VRE \)) was calculated by removing the absolute value term in Eq. (2).

**Results and discussion**

**Validation of altimetry data**

A time series of surface water level from the gauging station was used to validate the lake altimetry data. Over the period from September 1992 to May 2010, there were 202 total match-ups of Urmia Lake satellite and gauge water level data. Fig. 4 shows the scatter plot of the radar altimetry data against in-situ gauge records in Urmia Lake. Having a near unit slope of the regression line and a coefficient of determination equal to 0.99, radar altimetry data are in complete accord with the in-situ data. Nonetheless, the radar altimetry data slightly overestimate the actual water level of Urmia Lake at lower lake levels. As the water level of the lake increases to above 1276 m, the discrepancy between radar data and the gauge water level values decreases significantly. Additionally, variation of the lake water level during 1992–2010 was compared (Fig. 3). According to Fig. 4, long-term variation of the radar altimetry data matches well with the measured water levels in Urmia Lake.

In order to assess accuracy of radar altimetry data, the mean bias error (MBE), the mean absolute error (MAE) and the root mean square error (RMSE) were also calculated. Their values were 9.2, 17.4 and 22.8 cm, respectively. These relatively small error values confirm the validity of radar altimetry data in Urmia Lake. Accuracy of radar altimetry for calculation of a lake level varies from 2–3 cm for the large lake to 30–40 cm for small lakes (Crétaux et al., 2011). With RMSE value of 22.8 cm, the accuracy of Urmia Lake altimetry data falls within the valid range, although it is near the upper band of the error spectrum.

As displayed in Fig. 3, as the lake water level decreases, the discrepancy from the 45° line increases. Fig. 4 also shows that the disagreement between the radar and in-situ level data has increased since 2003. This is due to the fact that when the lake becomes drier the radar altimetry data contain larger errors as a result of bottom surface exposure. The presence of reflective surfaces on or adjacent (vegetation, islands, satellite track near lake shore, etc.) to the lake can affect the accuracy of altimetry data and lead to deviation from in-situ gauge measurements (Ayan, 2007; Berry et al., 2005). Reflective surfaces can bias the altimetric signal and cause the lake height to appear closer to the satellite. Hence, satellite radar altimeters may overestimate the lake level height. The closer the ground tracks are to the coasts, the larger error in the lake level height. In such cases precision is only at the decimeter level (Mercier et al., 2002).

For Urmia Lake, a number of satellite tracks are located near the lake shores. Thus, when re-tracking waveforms from the lake to the satellite altimeter, the reflected wave can be highly contaminated because the average time of the signal represents not only water surface (which is significantly smooth compared to the ground surface topography) but also the nearby ground topography. Moreover, as the lake desiccates, the bed, which might still be wet or covered by thin salt crusts, acts like a reflective surface and causes high backscatter (Fig. 5). Additionally, increase in the lake’s salinity beyond the saturation

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**Fig. 2.** Flowchart of the process followed to derive the reference V–A–L relationships of Urmia Lake using multi-source satellite data.

**Fig. 3.** Scatter plot of radar altimetry data against in-situ gauge level data in Urmia Lake.
level generates small salt scales, which float on the lake surface (Fig. 5). Salt scales can increase the backscattering of radar pulses, which in turn increases the measurement error. Consequently, the water level calculated by altimeter overestimates the actual level of Urmia Lake.

Validation of the HYDROWEB area data

The HYDROWEB provided rating curve for Urmia Lake has been calculated based on un-validated water level data and thus its application to calculate the lake area data may cause error. Therefore, it is essential to modify Urmia Lake rating curve using the validated water level data. Through use of the lake area data acquired from high-resolution satellite images and their concurrent validated water level data a modified rating curve was developed and applied to calculate the lake area time-series. Use of validated water levels instead of pure altimetry data leads to a minor change in the rating curve developed by the HYDROWEB and thus the calculated area data.

The modified rating curve leads is highly correlated with the Hydroweb rating curve and so there is excellent agreement between the area estimates based on either curve ($y = 1.00x - 72.63, R^2 = 1.00$). A negative bias of 72 km$^2$ indicates that the HYDROWEB surface areas are slightly larger than those calculated from the modified hypsometric curve. This is because of the higher accuracy of the modified hypsometric curve, which has been developed based on the validated water level data.

Area–elevation relationship

Given the validated water level radar-derived data and simultaneous surface areas, the $A$–$L$ relationship can be developed for Urmia Lake. The scatter plot of Urmia Lake water level versus its surface area is displayed in Fig. 8. A third order polynomial trend line can be well ($R^2 = 0.98$) fitted to data as follows:

$$A = 4.67 \times 10^9 - 11.02 \times 10^6 L + 8.68 \times 10^3 L^2 - 2.28 L^3$$ (3)

where $L$ is the lake level in meters and $A$ is the lake surface area in km$^2$.

Surface areas obtained from other Urmia Lake mapping studies were used to assess the accuracy of the proposed $A$–$L$ relationship (Table 1). Given the acquisition date of satellite images, the concurrent water level data are obtained from the gauge records and substituted in the $A$–$L$ equation. Then, results are compared with those retrieved from satellite images (Table 1). Results reveal that the estimated areas correlate well ($R^2 = 0.86$) with satellite-derived areas, having a RMSE of 222 km$^2$ (4.5%) and MBE of 310 km$^2$.

In the absence of accurate bathymetric data, the proposed $A$–$L$ equation (Eq. (3)) can be applied for Urmia Lake. However, below the range of altimetry data for water level (1271.45 m), uncertainty of using the $A$–$L$ equation increases. For example, fitting either a second or a third order polynomial to describe the $A$–$L$ relationship within the available data set results in a relatively small difference (Fig. 6). Nevertheless, in the lower part of the curve (equal to lower isobaths) the difference becomes more apparent.

For a given lake, slopes of the $A$–$L$ and $V$–$L$ curves highly depend on its morphology. In wide shallow riverine lakes, where initial drainage causes losses in the area and volume, the slope of the volume–depth curve at its lower part is milder compared to the relatively steep ‘U’ shaped lakes (Young et al., 2003). This should be considered when fitting an $A$–$L$ curve for Urmia Lake, which is a wide shallow lake. Fig. 8 shows that in comparison to the second order fit, the third order fit has a milder slope. Therefore, the third order fit seems to be a better representative of the $A$–$L$ relationship for Urmia Lake.

Considering the elevation of Urmia Lake bed, the second order fit gives 1268.92 m as the extreme bottom level of the lake, while the third order fit predicts it to be 1264.43 m. To verify the calculated values for the elevation of the lake bed, it is necessary to use field data. One of the few bathymetric studies of Urmia Lake is the survey conducted in 15 Oct 2002 on the western part of the lake by Alipour (2006). In that study the deepest part of the lake, located in the north section, was reported to be 9 m (Alipour, 2006). According to the gauge data, water level of the lake at the time of that study was 1273.29 m. Consequently, the elevation of Urmia Lake at the deepest
part (base level) is 1264.29 m. The third order fit was chosen to represent the A–L equation for Urmia Lake because it provides a better estimate for the level of the lake bed compared to the second order fit. However, due to the lack of topographic data below the minimum observed water level by the altimeters, uncertainty of using the A–L equation at low water levels is high. These errors typically arise from small topographic variations (micro-topography) in the bottom of Urmia Lake, which may not be apparent without a detailed bathymetric survey.

Lake volume characteristics

To derive the volume–elevation (or depth) relation for Urmia Lake three methods were used: Integration of the A–L equation, a power-function model and a simple truncated pyramid formula. Next, assuming the V–L relation derived from integration of the A–L equation is the most accurate (in the range of the lake altimetry data), the next two equations were validated based on it.

From area–elevation relationship

If the lake water level rises by a small amount $\Delta L$, the resulting volume change in the lake $\Delta V$ is equal to $A \times \Delta L$. Therefore, $V$ at any level $L$ is given by:

$$ V = \int_0^h A(\omega) \, d\omega $$

(4)

where $\omega$ is a dummy variable of integration, and $L_0$ is the lake base level (equal to elevation at the deepest point of the lake). This relationship between the $V$ and $A$ is applicable to all lakes and wetlands with a horizontal water surface (Hayashi and van der Kamp, 2000).

Substituting Eq. (3) in to Eq. (4) enables expressing the lake volume as a 4th order function of water level:

$$ V = -1.48 \times 10^9 + 4.67 \times 10^6 L - 5.51 \times 10^3 L^2 + 2.89 L^3 - 5.70 \times 10^{-4} L^4 $$

(5)

where the $L$ (m) and $V$ (km$^3$) are the water level and volume, respectively. Eq. (5), will be referred to as the reference $V$–L model henceforth.

As discussed for the A–L equation, because estimates of the lowest lake volumes are based on extrapolation of the reference $V$–L values, they are subject to a large amount of error. Unlike the A–L curve, these errors propagate in calculating the lake volume even in the high water levels. Such errors can only be eliminated through costly and labor-intensive topographic surveys. Therefore, Eqs. (3) and (5) can be applied in the range of radar altimetry data ($L \geq 1271.74$ m) with more confidence.

From the Nilsson’s power-function model

One of the equations used to describe the $V$–L relationship of Urmia Lake is a generalized formula proposed by Nilsson et al. (2008). This equation requires the maximum or reference pool area obtained from aerial photographs or other data sources and the associated maximum pool depth corresponding to that area. The equation is in the form of a simple power function relating the lake volume ($V$) to the lake pool depth ($h$) using a single dimensionless parameter ($m$) as follows:

$$ V_{pm}(h) = \left( \frac{A_{max} h_{max}}{m} \right)^m $$

(6)

where $V_{pm}(h)$ (km$^3$) is the lake volume corresponding to the lake pool at depth $h$ (m); $A_{max}$ (km$^2$) is the maximum or reference pool area corresponding to the maximum lake pool depth $h_{max}$ (m); and $m$ is a new dimensionless fitting parameter that describes the lake volume–depth ($V$–$h$) geometric relationship. Depending on the value of the shape parameter ($m$) various geometries can be described. For instance $m = \infty$ in Eq. (6) produces a vertical line at the maximum pool depth, representing cylindrical volume; $m = 1$ produces a planar curve, and $0 < m < 1$ and $1 < m < \infty$ produce convex and concave $V$–$h$ curves, respectively. The best performance of the $V$–$h$ power model is on circular, bowl-shaped wetlands and lakes. In other words, the shape factor represents deviation of the lake shape from a circular bowl (Nilsson et al., 2010).

The $V$–$h$ power function was tested in five cypress wetlands, five marsh wetlands, and 17 lakes located in the United States. For each depression, the dimensionless shape parameter ($m$) was calculated using available wetland volume data, so that the root-mean-square error (RMSE) between the reference (GIS-derived, observed, or reported volumes) and the model generated volumes can be minimized (Nilsson et al., 2010).
The concurrent dataset of the lake depth and volume in the range of radar altimetry data were used to derive the shape function for Urmia Lake. Depth data of the lake were obtained from the validated altimetry data and the measured base level of Urmia Lake. Afterward, the concurrent volumes were calculated using Eq. (5) and served as the reference values of the lake volume in Eq. (2). Furthermore, based on the altimetry data, maximum depth and area of the lake were found to be 13.34 m and 5497 km², respectively. Then, using a spreadsheet iterative solver (Microsoft Excel Solver) the dimensionless parameter (m) was fitted, so that the RMSE between the V–h power model and the fourth order polynomial was minimized. The resulting V–L relationship can be expressed as:

\[ V_{\text{PM}} = 0.28 \ (L - L_0)^{1.9} \]  

where \( V_{\text{PM}} \) is the estimated volume and the reference value of the lake volume was the lake geometry is approximated as a truncated pyramid with a floor is not horizontal and deepens toward the north side. Therefore, it is essential to first determine the lake mid-level. Utilizing simultaneous datasets of the lake level, area and volume obtained from altimetry data and the reference V–A–L models, an iterative procedure was used to find the mid-level (\( L_\text{m} \)) using the formula of a truncated pyramid and a complete pyramid, respectively. Therefore, it is essential to first determine the lake mid-level. Utilizing simultaneous datasets of the lake level, area and volume obtained from altimetry data and the reference V–A–L models, an iterative procedure was used to find the mid-level. Initially, a desired value was chosen for \( L_\text{m} \), as the first estimate and was substituted into the truncated pyramid formula to get the lake volume. Then, through trial and error, the first guess was modified until the difference between the estimated volume and the reference value of the lake volume was minimized. As a result, \( L_\text{m} \) was equal to 1269.39 m, and the lake volume at any desired state \( i \) is estimated by the following conditional equation:

\[
\begin{align*}
    V_i &= \frac{1}{3000} (L_i - L_\text{m}) (A_\text{m} + \sqrt{A_\text{m} A_\text{max}}) + \frac{1}{3000} (L_\text{m} - L_0) A_\text{m} \quad L_i > L_\text{m} \\
    V_i &= \frac{1}{3000} (L_i - L_0) A_i \quad L_i \leq L_\text{m}
\end{align*}
\]  

\[ V_{\text{PM}} = 0.28 \ (L - L_0)^{1.9} \]  

where \( L_0 \) and \( L_\text{m} \) are the lake base and middle levels; \( A_\text{m} \) is the lake area at the mid-level which can be calculated from Eq. (3); \( L_0 \), \( A_i \) and \( V_i \) are the lake water level (m), area (km²) and volume (km³) at the state of \( i \).

### Accuracy of the approximate V–L equations

#### Accuracy of the Nilsson-power model

After setting the shape parameter of the power-function model for Urmia Lake, accuracy of the equation was determined by the RMSE and the absolute relative volumetric error (VARE). For Urmia Lake, the RMSE and VARE are 1.5% and 2.9%, respectively. These values can be compared to values obtained from 17 lakes studied by Nilsson et al. (2008). In that study, RMSE and VARE values varied from 0.4 to 6.8% and 1.3 to 22.8%, respectively. Thus, the computed error values for Urmia Lake fall within the lower error bounds stated by Nilsson et al. (2008). These small values of error statistics show that the power model has a satisfactory performance in characterizing the V–L relation for Urmia Lake. Moreover, the calculated relative volumetric error (VRE = 2.7%) indicates little bias in over-prediction of Urmia Lake volume.

#### Accuracy of the truncated pyramid model

To derive the V–L equation of a lake, one can approximate the lake geometry to a frustum of a pyramid using the following formula:

\[ V_{\text{TPM}} = \frac{1}{3} \left( L_2 - L_1 \right) \left( A_1 + A_2 + \sqrt{A_1 A_2} \right) \]  

where \( V_{\text{TPM}} \) is the volume of the stored water between state 2 and 1 (km³); \( L_2 \) and \( L_1 \) are the lake water levels (m); and \( A_1 \) and \( A_2 \) are the lake water surface area at two desired states 2 and 1, respectively.

According to the survey conducted by Alipour (2006), the lake floor is not horizontal and deepens toward the north side. Therefore, the lake geometry is approximated as a truncated pyramid with a northward inclined bed (Fig. 7). Because of this, the lake volume is calculated differently above and below the mid-level (\( L_\text{m} \)) using the formula of a truncated pyramid and a complete pyramid, respectively. Therefore, it is essential to first determine the lake mid-level. Utilizing simultaneous datasets of the lake level, area and volume obtained from altimetry data and the reference V–A–L models, an iterative procedure was used to find the mid-level. Initially, a desired value was chosen for \( L_\text{m} \), as the first estimate and was substituted into the truncated pyramid formula to get the lake volume. Then, through trial and error, the first guess was modified until the difference between the estimated volume and the reference value of the lake volume was minimized. As a result, \( L_\text{m} \) was equal to 1269.39 m, and the lake volume at any desired state \( i \) is estimated by the following conditional equation:

\[
\begin{align*}
    V_i &= \frac{1}{3000} (L_i - L_\text{m}) (A_\text{m} + \sqrt{A_\text{m} A_\text{max}}) + \frac{1}{3000} (L_\text{m} - L_0) A_\text{m} \quad L_i > L_\text{m} \\
    V_i &= \frac{1}{3000} (L_i - L_0) A_i \quad L_i \leq L_\text{m}
\end{align*}
\]

![Fig. 7. Schematic approximation of Urmia Lake geometry by a truncated pyramid with a northward inclined bed.](image)

![Fig. 8. Comparison of the power-function model proposed by Nilsson et al. (2008) with values obtained from the reference equation.](image)
Long-term variation of Urmia Lake area and volume

Utilizing the continuous gauged water level data and the reference V–A–L models, the long-term variation of Urmia Lake surface area and volume from 1965 to 2011 were examined. As shown in Fig. 10, surface area of Urmia Lake increased between 1965 and 1970 and changed slightly up to 1991. Thereafter, the lake area has increased and peaked in 1996. Since then the lake has steadily declined in areal extent until the end of May 2011. From 2003 on, the lake water level, area and volume fell below the minimum historical values observed during the past four decades.

To evaluate current state of the lake with respect to its desired condition described in the integrated management plan of Urmia Lake, the lake storage at the current level (1270.7 m) and its minimum ecological level (1274.1 m) were calculated from the reference V–L equation and compared. Results show that currently, Urmia Lake has a volume of 8.8 km$^3$ which means a 13.2 km$^3$ water deficit from its minimum ecologically desired condition. This huge water shortage is a symptom of the lake exposure to severe environmental stress which has never been experienced before. Accurate quantification of negative water balance in Urmia Lake through the analytical V–A–L relationships can help water managers and policymakers to make informed decisions about allocation and supply of the lake water requirements.

Summary and conclusions

This study investigated the volume–area–elevation characteristics of Urmia Lake in the absence of an accurate bathymetry map, utilizing remote sensing data and analytical equations. The lake water level data from radar altimetry were acquired and validated against the gauge data. Then, employing concurrent data of the lake water level and surface area, the V–A–L curves of Urmia Lake were developed. Afterward, two approximate models, a power model and a truncated pyramid model, were parameterized for Urmia Lake and their accuracy were evaluated. Results reveal that the power model over-predicts the volume of Urmia Lake, whereas the truncated pyramid model underestimates it. In addition, the power model is a more reliable approximation of Urmia Lake geometry compared with the truncated pyramid model.

Variations in the lake area and volume between 1965 and 2011 were examined using the developed reference V–A–L equations. Our analysis showed that the lake area and volume have dramatically declined in the last 5 decades. Currently, Urmia Lake faces a deficit of 13.2 km$^3$ below its minimum ecological level. This study clarifies the historical negative trends in the lake area and volume, which may have severe consequences without implementation of conservation measures. Moreover, appropriateness of remote sensing data as a cost-effective alternative to detailed bathymetric surveys was confirmed in this research.

One of the shortcomings of this study is neglecting the lake geometrical changes due to the salt precipitation. In a hyper-saline chemically-saturated lake, a decline in the water level is not necessarily an indicator of the actual volumetric changes because accumulation of the salts
effectively raises the lake bed. This results in the possible underestimation of the water loss (Lensky et al., 2005). Inclusion of water quality data, as a suggestion for future work, can improve accuracy of the V–A–L relations for Urmia Lake. Additionally, the effect of islands and the constructed causeway on the lake topography should be determined by detailed bathymetric surveys.

The three proposed approaches for deriving Urmia Lake V–A–L relations can also be applied to other large lakes and wetlands where accurate bathymetry maps are not available. However, having sufficient data about altimetry, surface area, and maximum depth/base level of lakes are still required. When available, gauge data should be used to validate altimetry data. Nevertheless, without validation of altimetry data, it is still possible to develop such models and assess them, albeit uncertainty in the derived equation remains unknown.

As an alternative, to develop accurate V–A–L predictions, it is required to have at least the upper 80% of the lake storage data (Nilsson et al., 2008). For Urmia Lake, since the water level has been declining to a large extent, altimetry data cover a wide range of the lake variation, and therefore, this condition can be met. However, it may not be the case for other lakes, particularly, those with a relatively stable water level over time. Therefore, to develop analytical models of a lake V–A–L, not only the length of satellite-derived data (water level and area) but also distribution of data compared to the lake maximum storage should be adequate.

This study demonstrated that accurately parameterized analytical models like the PM can suitably resemble the lake storage, while simple formulations such as the TPM are not accurate enough. As shown for Urmia Lake, performance of the TPM is not acceptable to be used for variation of lakes storage, particularly for lakes without a flat bed. Alternatively, where it is applicable, the empirical reference model should be applied. These findings can be used by HYDROWEB to modify the calculated time series of volume variations for lakes with sufficient data of water level and area.

Finally, as in the case of Urmia Lake through use of multi-source satellite data, accurate analytical models can be developed and parameterized for other large lakes given sufficient level and surface area data (e.g. those are currently available within the HYDROWEB database). Then, as recommended by Nilsson et al. (2010), future research can be conducted to see if general parametric formulas can be developed for lakes with similar characteristics. Results of such studies can provide insights to water resource engineers and hydrologists for estimating lake water storage where bathymetric data do not exist.

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