

Heat Budgets

LECTURE OVERVIEW: I outline how a heat budget can be developed for a well-mixed system. Emphasis is placed on characterizing the surface heat exchange between the water and the atmosphere.

We ended the previous lecture with a model of a thermally stratified lake. As we saw, thermal stratification can have a pronounced effect on water quality. Consequently, understanding heat and temperature modeling is an important facet of water-quality modeling.

The mathematical modeling of the transport and fate of heat in natural waters has been the subject of extensive study. Edinger et al. (1974) provide an excellent and comprehensive report of this research. Thomann and Mueller (1987) have summarized the fundamental approach as it relates to water-quality modeling.

Most of this work has been oriented toward evaluating cooling-water discharges and has dealt with systems such as cooling ponds and large rivers. Today there is a broadening of interest in temperature modeling that goes beyond the effects of point sources of heat such as power plants. Several problem contexts that involve temperature and heat are:

- Physical processes (such as thermal stratification) along with biological and chemical transformations are sensitive to temperature. Therefore to adequately characterize these other problems, there may be cases where an accompanying analysis of heat would be needed.
- There is growing interest in the diurnal temperature variations of shallow, turbulent streams that are commonly found in upland regions. Although these systems are sometimes subjected to anthropogenic heat loads, their response to natural forcing functions is also of interest. Physical modifications, such as channelization and riparian zone denudation, can have a pronounced effect on their thermal regimes.

Remedial measures have been proposed to reverse these effects (Oswald and Roth 1988). Mathematical models could prove useful in the evaluation of these modifications. In addition diurnal temperature variations are relevant to the modeling of the fate of pollutants in such systems. For example water-quality problems such as ammonia toxicity are sensitive to diurnal temperature variations.

- Temperature effects on biota, some of which are threatened or endangered, are directing attention toward heat management. Aside from the drainage-basin modifications noted above, reservoir releases can also have an impact on temperatures in tail waters.

For all these problems, the extensive body of theory concerning the fate of heat provides a basis for temperature models of streams, estuaries, and lakes. In this lecture I'll describe the part of this theory that relates to surface heat exchange.

30.1 HEAT AND TEMPERATURE

To this point we have focused on mass balances of substances in natural waters. In particular we have used concentration to provide a measure of the intensity of a pollutant. For a volume of water V , the concentration c is related to the mass m by

$$c = \frac{m}{V} \quad (30.1)$$

Recall from our discussion in Lec. 1 that mass is an extensive (that is, size-dependent) property whereas concentration is intensive (size-independent).

A similar relationship can be developed for heat,

$$T = \frac{H}{\rho C_p V} \quad (30.2)$$

where T = temperature

H = heat

ρ = density

C_p = specific heat

Thus heat is the extensive quantity whereas temperature is intensive (Fig. 30.1).

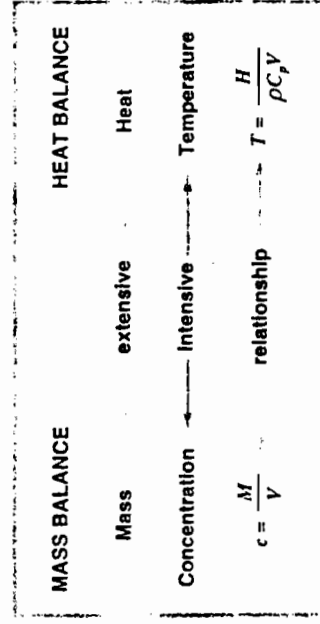


FIGURE 30.1

The analogy between mass and heat.

TABLE 30.1
Thermal units

	Symbol	MKS	CGS	English
Temperature	T	$^{\circ}\text{C}$ or K	$^{\circ}\text{C}$ or K	$^{\circ}\text{F}$ or $^{\circ}\text{R}$
Heat	H	joules	calories	Btu
Density	ρ	kg m^{-3}	g cm^{-3}	$\text{lb}_m \text{ft}^{-3}$
Volume	V	m^3	cm^3	ft^3
Specific heat	C_p	$\text{J (kg }^{\circ}\text{C)}^{-1}$	$\text{cal (g }^{\circ}\text{C)}^{-1}$	$\text{Btu (lb}_m \text{ }^{\circ}\text{F)}^{-1}$

Note that several systems of units can be used (Table 30.1) for the quantities in Eq. 30.2.

Observe that two parameters, density and specific heat, are included in Eq. 30.2 to reflect the type of substance that is being described. As in the following example, these parameters influence the temperature change that results from adding heat to different substances.

EXAMPLE 30.1. TEMPERATURE AND HEAT. Determine how much heat needs to be added to 1 m^3 of air, water, brick, and iron to induce a 1°C rise in temperature. Thermal properties of these substances (Kreith and Bohn 1986) are listed below:

Substance	Density (kg m^{-3})	Specific heat ($\text{J (kg }^{\circ}\text{C)}^{-1}$)
Dry air	1.164	1012
Water	998.2	4182
Common brick	1800	840
Cast iron	7272	420

Note: all values are for 20°C .

Solution: For air, Eq. 30.2 can be rearranged to compute

$$H = \rho C_p VT = 1.164 \frac{\text{kg}}{\text{m}^3} \left(1012 \frac{\text{J}}{\text{kg } ^{\circ}\text{C}} \right) (1 \text{ m}^3)(1^{\circ}\text{C}) = 1178 \text{ J}$$

The amount of heat for the other substances can be computed in a similar fashion. All the results can be tabulated as

Substance	Added heat (J)
Dry air	1178
Water	4.17×10^6
Common brick	1.51×10^6
Cast iron	3.05×10^6

Notice that, primarily due to its lower density, much less heat is required to induce a temperature rise in dry air than in the other substances. Also observe that although it is not as dense as the solids, more heat must be added to the water because of its high specific heat.

30.2 SIMPLE HEAT BALANCE

Just as a mass balance can be written for a volume of water, a *heat balance* can also be developed. As was the case for mass, the heat balance states that for a finite volume of water over a unit time period,

$$\text{Accumulation} = \text{sources} - \text{sinks} \quad (30.3)$$

A hypothetical completely mixed system is depicted in Fig. 30.2. For a finite time period, the heat balance for the system can be expressed as

$$\text{Accumulation} = \text{inflow} - \text{outflow} \pm \text{surface heat exchange} \quad (30.4)$$

In this balance there is a single inflow source that contributes heat. Although this is labeled "inflow" it represents both heat entering through tributary streams as well as point discharges. A loss is included due to heat leaving the system through the lake's outlet. Finally surface heat exchange represents the heat gained across the air-water interface due to interactions with the atmosphere. As indicated by the plus or minus sign, this term can be either a source or a sink, depending on the state of the lake and the atmosphere.

Other sources and sinks could have been included in the heat balance. For example exchange of energy with the sediments can be an important term in very shallow systems. However, since sediment-water exchange is not significant for most natural water bodies, we limit ourselves to the terms outlined in Fig. 30.2 and Eq. 30.4.

Although Eq. 30.4 has descriptive value, it cannot be used to predict water quality. To do this we now express each term as a function of measurable variables and parameters.

Accumulation. Accumulation represents the change of heat H in the system over time t .

$$\text{Accumulation} = \frac{\Delta H}{\Delta t} \quad (30.5)$$

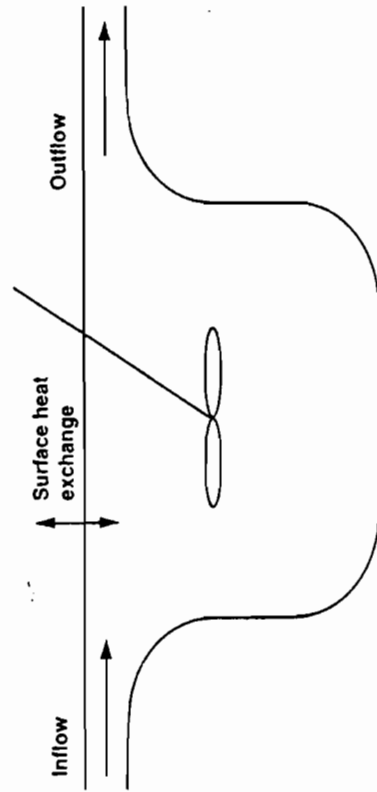


FIGURE 30.2

A heat balance for a well-mixed lake. The arrows represent the major sources and sinks of heat.

Substituting Eq. 30.2 into 30.5 yields

$$\text{Accumulation} = \frac{\Delta\rho C_p VT}{\Delta t} \quad (30.6)$$

In the present case we assume that the density and specific heat of water are relatively constant over the range of temperatures for the lake. We also assume that the lake's volume is constant. These assumptions allow us to bring the parameters outside the difference. Finally when Δt is made very small, Eq. 30.6 reduces to

$$\text{Accumulation} = \rho C_p V \frac{dT}{dt} \quad (30.7)$$

Thus heat accumulates as temperature increases with time (positive dT/dt) and diminishes as it decreases with time (negative dT/dt). For the steady-state case, heat remains constant ($dT/dt = 0$). Note that the units of accumulation (as with all other terms in the balance) are heat per time ($J d^{-1}$). This can be seen by inspecting the units of the individual terms in Eq. 30.7,

$$\rho C_p V \frac{dT}{dt} = \frac{\text{kg}}{\text{m}^3} \left(\frac{\text{J}}{\text{kg}^\circ\text{C}} \right) (\text{m}^3) \left(\frac{^\circ\text{C}}{\text{d}} \right) = \frac{\text{J}}{\text{d}} \quad (30.8)$$

Inflow. As was the case with mass, we lump all point and nonpoint sources of heat entering the lake's periphery into a single term,

$$\text{Inflow} = Q\rho C_p T_{in}(t) \quad (30.9)$$

where Q = volumetric flow rate of all water sources entering the system and $T_{in}(t)$ = average inflow temperature of these sources. For the present case we have assumed that flow is constant and that all the temporal variations in heat inputs are due to temporal variations in the inflow temperature.

Outflow. In our simple system (Fig. 30.2) heat is carried from the system by an outflow stream. The rate of heat transport can be quantified as a function of the outflow temperature T_o . However, because of our well-mixed assumption, the outflow temperature by definition equals the mid-lake temperature. $T_o = T$, and the outflow sink can be represented by

$$\text{Outflow} = Q\rho C_p T \quad (30.10)$$

Surface heat exchange. The net heat exchange across the lake's surface can be represented as a flux,

$$\text{Surface heat exchange} = A_s J \quad (30.11)$$

where A_s = lake's surface area (m^2) and J = surface heat flux ($J \text{ m}^{-2} \text{ d}^{-1}$), with a positive flux representing a heat gain.

Total balance. The terms can now be combined into the following heat balance for a well-mixed lake:

$$V\rho C_p \frac{dT}{dt} = Q\rho C_p T_{in}(t) - Q\rho C_p T + A_s J \quad (30.12)$$

EXAMPLE 30.2. HEAT BALANCE FOR A WELL-MIXED POND. A pond has the following characteristics:

- Volume = $50,000 \text{ m}^3$
- Surface area = $25,000 \text{ m}^2$
- Mean depth = 2 m
- Inflow = outflow = $7500 \text{ m}^3 \text{ d}^{-1}$

The pond's inflow has a temperature of 20°C . The net heat gain from the atmosphere is $250 \text{ cal cm}^{-2} \text{ d}^{-1}$. If there is no other heat exchange, calculate the steady-state temperature.

Solution: First, the surface heat flux can be converted to the proper units,

$$J = 250 \frac{\text{cal}}{\text{cm}^2 \text{ d}} \left(\frac{10,000 \text{ cm}^2}{\text{m}^2} \right) \left(\frac{\text{J}}{0.2388 \text{ cal}} \right) = 1.047 \times 10^7 \text{ J m}^{-2} \text{ d}^{-1}$$

Then, Eq. 30.12 can be solved for

$$T = T_{in} + \frac{A_s J}{Q\rho C_p} = 20 + \frac{25,000(1.047 \times 10^7)}{7500(998.2)(4182)} = 20 + 8.36 = 28.36^\circ\text{C}$$

Thus the temperature is increased 8.36°C by the surface heat flux.

In the previous example, we treated atmospheric heat exchange as a single entity. In fact, as described next, it consists of a number of different mechanisms.

30.3 SURFACE HEAT EXCHANGE

As depicted in Fig. 30.3, surface heat exchange can be modeled as a combination of five processes. Note that the processes can be clustered in two ways. First, we can distinguish between radiative and nonradiative mechanisms. *Radiation* refers to energy that is transmitted in the form of electromagnetic waves and thus does not

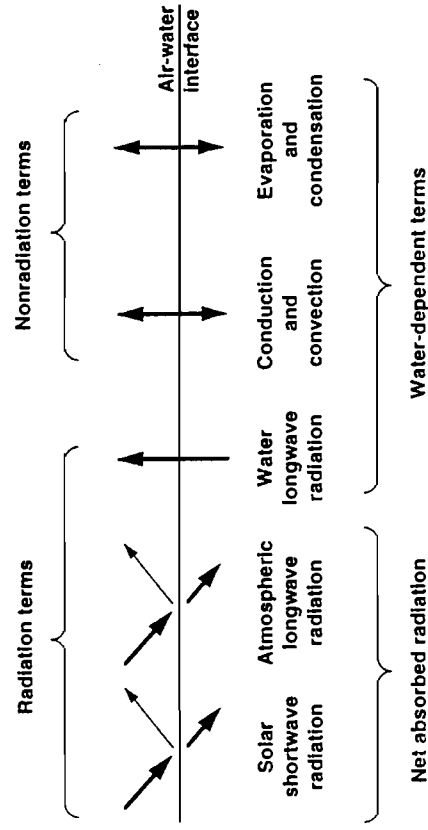


FIGURE 30.3
The components of surface heat exchange.

depend on matter for its transmission. In contrast, processes such as conduction and evaporation depend on the motion of molecules.

The second way in which the mechanisms can be divided is based on whether they are dependent on the temperature of the water. This division relates to whether the process acts as a model forcing function or includes the dependent variable: water temperature. Processes such as solar radiation and atmospheric longwave radiation are independent of the condition of the water. Hence these terms act as forcing functions. As labeled in Fig. 30.3, they constitute the net radiation absorbed by the water. In contrast, although they are dependent on external conditions (air temperature, moisture, wind speed, etc.), processes such as evaporation are also a function of water temperature.

The total surface heat flux can be represented as

$$J = J_{in} + J_{an} - (J_{br} + J_c + J_e) \tag{30.13}$$

Net absorbed radiation Water-dependent terms

- where J_{in} = net solar shortwave radiation
- J_{an} = net atmospheric longwave radiation
- J_{br} = longwave back radiation from the water
- J_c = conduction
- J_e = evaporation

In the following sections we outline how these terms are quantified. Before doing this we must provide some background information on two subjects that have a strong bearing on some of the terms: the Stefan-Boltzmann law and atmospheric moisture.

30.3.1 Stefan-Boltzmann Law

All objects with a temperature above absolute zero emit radiation. The higher the object's temperature, the shorter the wavelength of the emission and the greater the quantity of energy emitted per unit of surface area.

The Stefan-Boltzmann law states that the maximum rate of radiation emitted per unit area of surface can be expressed mathematically as

$$J_{rad} = \epsilon \sigma T_a^4 \tag{30.14}$$

- where T_a = absolute temperature (K)
- σ = the Stefan-Boltzmann constant [$= 11.7 \times 10^{-8} \text{ cal (cm}^2 \text{ d K}^4)^{-1}$]
- ϵ = emissivity of the radiating body

The emissivity is a correction factor to account for the fact that the body is not a perfect emitter of radiation. The law is named after two Austrians, Josef Stefan, who discovered it experimentally, and Ludwig Boltzmann, who derived it theoretically.

30.3.2 Atmospheric Moisture

Recall from Example 30.1 that dry air and water have very different densities and specific heats. Consequently something must be done to account for the fact that air

can have a wide range of water content. The range of water content can be represented by the percent relative humidity R_h , which is the ratio of the air's actual water content to its maximum possible level at the same temperature.

$$R_h = 100 \frac{e_{air}}{e_{sat}} \tag{30.15}$$

where e_{air} = vapor pressure of the air (mmHg) and e_{sat} = saturation vapor pressure (mmHg). The latter quantity is a function of temperature and can be calculated by (modified for units from Raudkivi 1979)

$$e_{sat} = 4.596 e^{\frac{17.27T}{237.3+T}} \tag{30.16}$$

Figure 30.4 shows the resulting dependency of saturation on temperature.

Finally the *dew-point temperature* is the temperature at which a mass of air just becomes saturated when cooled at a constant pressure and water content. The name makes sense if you think about the situation that occurs at night. At sundown, if there is no precipitation, the air will likely have a water content below saturation. After the sun goes down, the air usually cools until it reaches its lowest value just before sunrise. As the air cools (assuming that the pressure and the moisture content remain relatively constant), the saturation level decreases according to Eq. 30.16. If the process continues, there will come a point at which the moisture content will equal the saturation value. At this point the air cannot hold additional water and dew forms.

Inspection of Fig. 30.4 indicates that the air vapor pressure is equivalent to the saturation pressure corresponding to the dew-point temperature,

$$e_{air} = 4.596 e^{\frac{17.27T_d}{237.3+T_d}} \tag{30.17}$$

These concepts are important since they relate to quantifying the heat loss due to evaporation. For example Dalton (1802) proposed that evaporation was proportional to the difference between the vapor pressure of the air and the saturated vapor pressure calculated at the temperature of the water surface T_s :

$$J_e = -f(U_w)(e_s - e_{air}) \tag{30.18}$$

where e_s = saturation vapor pressure corresponding to the surface temperature of

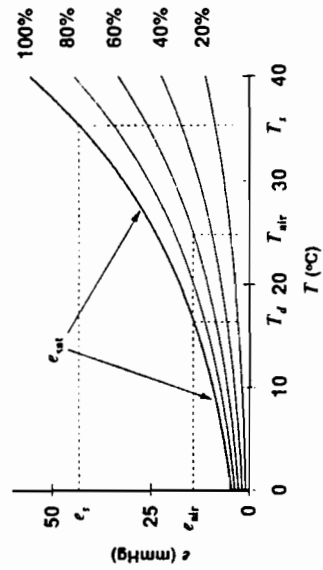


FIGURE 30.4
The dependency of saturation vapor pressure on temperature. Lines of constant relative humidity are displayed. Also shown are the vapor pressures corresponding to the water, air, and dew-point temperature from Example 30.3.

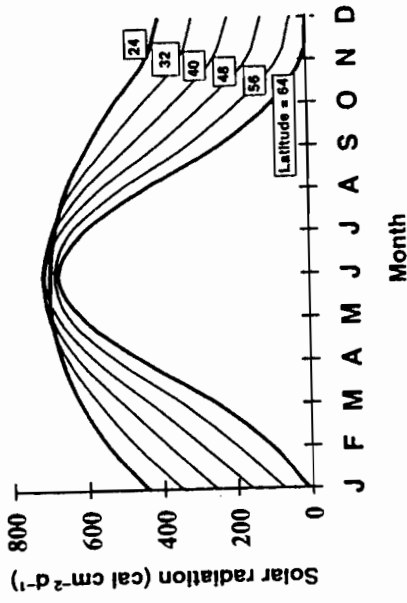


FIGURE 30.5
Daily totals of solar radiation for different latitudes as a function of time of year (from Kreider 1982).

- **Scattering and absorption.** Once sunlight enters the atmosphere it is scattered by dust, reflected by clouds, and absorbed by atmospheric gases.
- **Reflection.** Upon reaching the water, a portion of the radiation will be scattered by reflection at the surface. The percent of radiation returning from a surface compared to that which strikes it is formally referred to as the surface's *albedo*. As depicted in Fig. 30.7, for a flat water surface the fraction reflected is significant only when the sun is low in the sky. In addition the condition of the sky (clear/overcast) and the water surface (flat/waves) can affect reflection.
- **Shading.** Some streams are located in deep canyons or are lined by tall trees. In such cases the resulting shading can greatly decrease solar radiation.

Solar radiation is usually obtained from direct measurements or equations. The latter are a function of factors such as time, position, and cloud cover. Some of the commonly used algorithms are summarized in a number of references (Eagleson 1970, Bras 1990, Brown and Barnwell 1987, etc.)

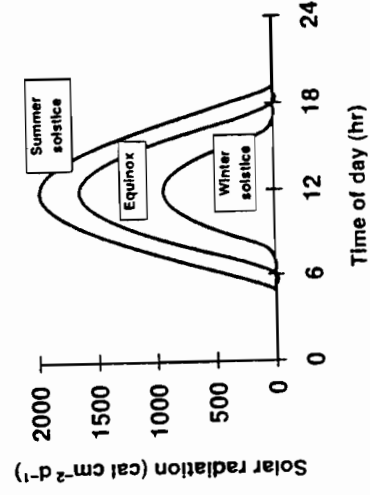


FIGURE 30.6
Diurnal trends of clear-sky solar radiation on a horizontal surface at latitude 40°N on the summer solstice (June 21), the winter solstice (Dec. 21), the vernal equinox (Mar. 21), and the autumnal equinox (Sep. 21) (from Kreider 1982).

the water ($^{\circ}\text{C}$) and $f(U_w)$ = a transfer coefficient that depends on the wind speed measured a fixed distance above the water surface.

It is unlikely that you will be provided with a value for the vapor pressure in the air above a water body. It is more likely that you will be able to obtain estimates for air temperature, dew-point temperature, and/or relative humidity. As illustrated by the following example, all you require is two of these quantities to calculate the remaining value as well as the vapor pressure of the air.

EXAMPLE 30.3. RELATIVE HUMIDITY, DEW POINT, AND AIR TEMPERATURE. The atmosphere above a lake has an air temperature of 25°C , a relative humidity of 60%, and the surface water temperature is 35°C . Use this information to determine (a) the air vapor pressure and the dew-point temperature and (b) whether evaporation or condensation will occur.

Solution: (a) Equation 30.16 can be used to compute the saturation vapor pressure.

$$e_{\text{sat}} = 4.596e^{\frac{17.27(25)}{237.3+25}} = 23.84 \text{ mmHg}$$

Therefore Eq. 30.15 can be used to determine the air vapor pressure.

$$e_{\text{air}} = \frac{R_v e_r}{100} = \frac{60(23.84)}{100} = 14.3 \text{ mmHg}$$

Then this value can be substituted into Eq. 30.17,

$$14.3 = 4.596e^{\frac{17.27T_d}{237.3+T_d}}$$

which can be solved for

$$T_d = \frac{237.3}{\frac{17.27}{\ln(14.3/4.596)} - 1} = 16.7^{\circ}\text{C}$$

(b) The saturation vapor pressure corresponding to the surface temperature of the water can be computed as

$$e_s = 4.596e^{\frac{17.27(35)}{237.3+35}} = 31.93 \text{ mmHg}$$

Because $e_r > e_{\text{air}}$, Dalton's law indicates that heat (and water) will be lost from the lake by evaporation.

30.3.3 Net Absorbed Radiation

Two sources of radiation add energy to a water body: shortwave radiation from the sun and longwave radiation from the atmosphere.

Solar shortwave radiation. The magnitude of solar radiation is dependent on several factors:

- **Solar altitude.** This will vary depending on the date, time of day, and location on the earth's surface. Figure 30.5 shows seasonal trends of solar radiation for different values of latitude. Figure 30.6 depicts the daily trend for various days of the year.

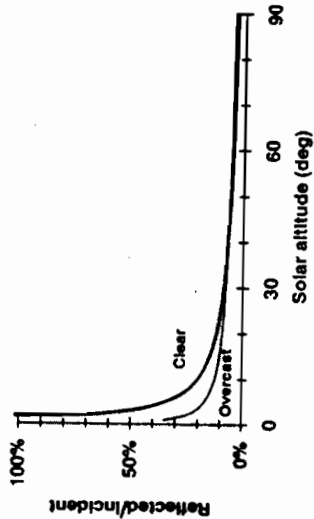


FIGURE 30.7
Ratio of reflected to incident solar radiation (albedo) as a function of solar altitude. Both clear and overcast sky conditions are shown (Brown and Barnwell 1987).

Atmospheric longwave radiation. The atmosphere itself emits longwave radiation. This gain can be represented as a modification of the Stefan-Boltzmann law,

$$J_{an} = \sigma(T_{\text{air}} + 273)^4 \left(A + 0.031\sqrt{e_{\text{air}}} \right) (1 - R_L) \quad (30.19)$$

Stefan-Boltzmann law Atmospheric attenuation Reflection

where σ = the Stefan-Boltzmann constant = $4.9 \times 10^{-3} \text{ J (m}^2 \text{ d K}^4)^{-1}$ or $11.7 \times 10^{-8} \text{ cal (cm}^2 \text{ d K}^4)^{-1}$

T_{air} = air temperature ($^{\circ}\text{C}$)

A = a coefficient (0.5 to 0.7)

e_{air} = air vapor pressure (mmHg)

R_L = reflection coefficient

The reflection coefficient is generally small (≈ 0.03).

30.3.4 Water-Dependent Terms

Water longwave radiation. The back radiation from the water surface can also be represented by the Stefan-Boltzmann law,

$$J_{br} = \epsilon\sigma(T_s + 273)^4 \quad (30.20)$$

where ϵ = emissivity of water (approximately 0.97) and T_s = water surface temperature. The emissivity is a correction factor that accounts for the fact that the water is not a perfect emitter of radiation.

Conduction and convection. To this point all the terms have involved radiation. We now turn to the last two mechanisms: convection/conduction and evaporation/condensation. In both cases heat transfer is linked to matter.

Conduction is the transfer of heat from molecule to molecule when matter of different temperatures come into contact. As such, it is analogous to the diffusive transport previously described in Lec. 8. **Convection** is heat transfer that occurs due

to mass movement of fluids. Both can occur at the air-water interface and can be described by

$$J_c = c_1 f(U_w)(T_s - T_{\text{air}}) \quad (30.21)$$

where c_1 = Bowen's coefficient ($\approx 0.47 \text{ mmHg } ^{\circ}\text{C}^{-1}$). The term $f(U_w)$ defines the dependence of the transfer on wind velocity over the water surface, where U_w is the wind speed measured a fixed distance above the water surface. Many relationships exist to define the wind dependence. Bras (1990) provides a review. Edinger et al. (1974) have suggested the relationship proposed by Brady, Graves, and Geyer (1969),

$$f(U_w) = 19.0 + 0.95U_w^2 \quad (30.22)$$

where the wind speed is measured in m s^{-1} at a height of 7 m above the water surface.

Evaporation and condensation. The heat loss due to evaporation can be represented by Dalton's law,

$$J_e = f(U_w)(e_s - e_{\text{air}}) \quad (30.23)$$

where e_s = saturation vapor pressure at the water surface and e_{air} = vapor pressure in the overlying air (mmHg).

30.3.5 Total Heat Budget

The individual terms developed in the previous paragraphs can now be incorporated into Eq. 30.13,

$$J = \underbrace{J_{sn}}_{\text{Net solar}} + \underbrace{\sigma(T_{\text{air}} + 273)^4}_{\text{Atmospheric longwave}} \left(A + 0.031\sqrt{e_{\text{air}}} \right) (1 - R_L) - \underbrace{\epsilon\sigma(T_s + 273)^4}_{\text{Water longwave}} - \underbrace{c_1 f(U_w)(T_s - T_{\text{air}})}_{\text{Conduction}} - \underbrace{f(U_w)(e_s - e_{\text{air}})}_{\text{Evaporation}} \quad (30.24)$$

This surface heat flux can then be substituted into the heat balance for the well-mixed lake (Eq. 30.12). As described next, the resulting equation then provides a basis for predicting water temperature as a function of heat loads and atmospheric conditions.

30.4 TEMPERATURE MODELING

Now that we have developed a mechanistic model of surface heat exchange, the total heat balance can be used to predict the temperature of a natural water. For simplicity the following discussion focuses on the simple completely mixed lake introduced at the beginning of this lecture. It should be noted that the approach can be easily extended to other bodies of water such as streams and estuaries.

30.4.1 Steady-State

At steady-state a heat balance for the well-mixed lake in Fig. 30.2 can be written as

$$0 = \frac{\rho C_p Q T_{in}}{A_s} + J_{in} + \sigma(T_{air} + 273)^4 (A + 0.031 \sqrt{e_{air}}) (1 - R_L) - \frac{\rho C_p Q T_s}{A_s} - \epsilon \sigma(T_s + 273)^4 - c_1 f(U_w)(T_s - T_{air}) - f(U_w)(e_s - e_{air}) \quad (30.25)$$

EXAMPLE 30.4. STEADY-STATE HEAT BALANCE FOR A WELL-MIXED LAKE.

A pond has the following characteristics:

- Volume = 250,000 m³
- Surface area = 25,000 m²
- Inflow = outflow = 7500 m³ d⁻¹

The pond's inflow has a temperature of 10°C. In addition, it is subject to the following meteorological conditions:

- Net solar radiation = 300 cal cm⁻² d⁻¹
- Air temperature = 25°C
- Dew-point temperature = 16.7°C
- Wind speed = 3 m s⁻¹
- Relative humidity = 60%

Calculate the steady-state temperature.

Solution: Each term in the steady-state heat balance (Eq. 30.25) can be dealt with separately.

Inflow:

$$\frac{\rho C_p Q T_{in}}{A_s} = \frac{1(1)7500 \times 10^6(10)}{250 \times 10^6} = 300 \text{ cal cm}^{-2} \text{ d}^{-1}$$

Atmospheric longwave: This term requires estimates of A and e_{air} . We will arbitrarily choose a value of $A = 0.6$ as the midpoint of the range suggested previously. The vapor pressure of the air can be computed as

$$e_{air} = 0.6 \left(4.596 e^{\frac{17.27(25)}{237.3+25}} \right) = 14.3 \text{ mmHg}$$

These values can be substituted along with other parameters to compute

$$\begin{aligned} \sigma(T_{air} + 273)^4 (A + 0.031 \sqrt{e_{air}}) (1 - R_L) \\ = 11.7 \times 10^{-8} (25 + 273\text{K})^4 (0.6 + 0.031 \sqrt{14.3}) (1 - 0.03) = 642 \text{ cal cm}^{-2} \text{ d}^{-1} \end{aligned}$$

Outflow:

$$-\frac{\rho C_p Q T_s}{A_s} = -\frac{0.9982(1)7500 \times 10^6}{250 \times 10^6} T_s = -30T_s$$

Water longwave:

$$-\epsilon \sigma(T_s + 273)^4 = -0.97(11.70 \times 10^{-8})(T_s + 273)^4 = -11.35 \times 10^{-8}(T_s + 273)^4$$

Conduction: First, we must compute the wind effect (Eq. 30.22),

$$f(U_w) = 19.0 + 0.95(3)^2 = 27.55$$

This result can be substituted along with other values to compute the conduction losses,

$$-c_1 f(U_w)(T_s - T_{air}) = -0.47(27.55)(T_s - 25) = -12.95(T_s - 25)$$

Evaporation: We must first determine the vapor pressure for the water,

$$e_s = 4.596 e^{\frac{17.27 T_s}{237.3+T_s}}$$

This result, along with other parameters, can be substituted into Eq. 30.18,

$$-f(U_w)(e_s - e_{air}) = -27.55 \left(4.596 e^{\frac{17.27 T_s}{237.3+T_s}} - 14.3 \right)$$

The individual terms can now be consolidated into the total heat budget,

$$0 = 300 + 300 + 642 - 30T_s - 11.35 \times 10^{-8}(T_s + 273)^4 - 12.95(T_s - 25) - 27.55 \left(4.596 e^{\frac{17.27 T_s}{237.3+T_s}} - 14.3 \right) - J_{net} = -520 \text{ cal cm}^{-2} \text{ d}^{-1}$$

This nonlinear equation can be solved numerically for $T_s = 17.3^\circ\text{C}$. This value can then be substituted back into the heat budget to assess the relative magnitude of the individual terms. The results are shown below:

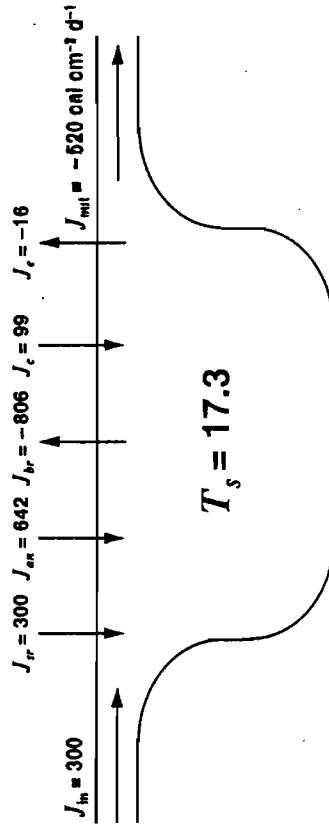


FIGURE E30.4

30.4.2 Time Variable

The surface heat exchange (Eq. 30.24) can also be incorporated into Eq. 30.12 to derive a time-variable heat budget,

$$\begin{aligned} \frac{dT_s}{dt} = \frac{Q}{V} T_m + \frac{J_{in}}{\rho C_p H} + \frac{\sigma(T_{air} + 273)^4 (A + 0.031 \sqrt{e_{air}}) (1 - R_L)}{\rho C_p H} \\ - \frac{Q}{V} T_s - \frac{\epsilon \sigma(T_s + 273)^4}{\rho C_p H} - \frac{c_1 f(U_w)(T_s - T_{air})}{\rho C_p H} - \frac{f(U_w)(e_s - e_{air})}{\rho C_p H} \end{aligned} \quad (30.26)$$

EXAMPLE 30.5. TIME-VARIABLE HEAT BALANCE FOR A WELL-MIXED LAKE. Compute the annual heat budget for the same pond used in Example 30.4. Values for meteorological variables have been provided (Table E30.5):

TABLE E30.5
Thermal units

Month	Solar radiation ($\text{cal cm}^{-2} \text{d}^{-1}$)	Air temperature ($^{\circ}\text{C}$)	Dew-point temperature ($^{\circ}\text{C}$)	Wind speed (km hr^{-1})
Jan	169	8.3	2.8	11.6
Feb	274	9.0	3.3	11.7
Mar	414	13.5	4.9	16.4
Apr	552	13.9	4.0	15.6
May	651	21.8	5.3	16.6
Jun	684	24.7	7.8	16.7
Jul	642	29.4	11.8	12.7
Aug	537	26.6	11.5	11.7
Sep	397	24.9	7.7	14
Oct	259	15.0	6.8	12.9
Nov	160	9.7	6.5	14.8
Dec	127	6.6	2.4	11.6

Assume that the volume and flows are constant and that the inflow temperature is fixed at a constant level of 10°C .

Solution: Before developing the solution, we can plot the meteorological variables (Fig. E30.5-1):

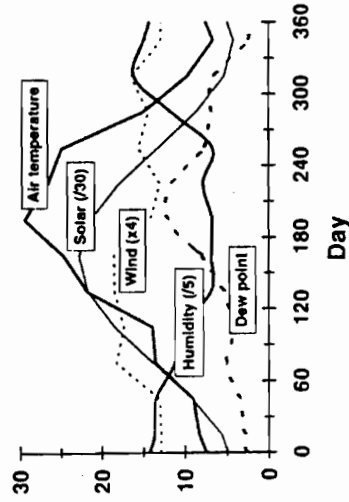


FIGURE E30.5-1
Meteorological variables.

Notice that we have computed and displayed the relative humidity along with the other meteorological variables. The peak solar radiation occurs on day 167 (mid-June) whereas the peak air and dew-point temperatures fall about a month later on day 198 (mid-July). High winds blow in the spring and to a lesser extent in the fall. Finally the highest relative humidities take place in the winter months.

Equation 30.26 can now be integrated with a method such as the fourth-order Runge-Kutta approach described in Lec. 7. The results are displayed in Fig. E30.5-2. We have included the solar radiation, as well as the inflow and the air and dew-point temperatures, on the plot for comparative purposes. Notice how the peak water temper-

ature occurs at about the same time of year as the peak air temperature. In contrast the peak solar radiation occurs about a month earlier.

Finally Fig. E30.5-3 shows the individual terms of the heat balance in flux units. Observe that the largest gains and losses are due to atmospheric and water radiation.

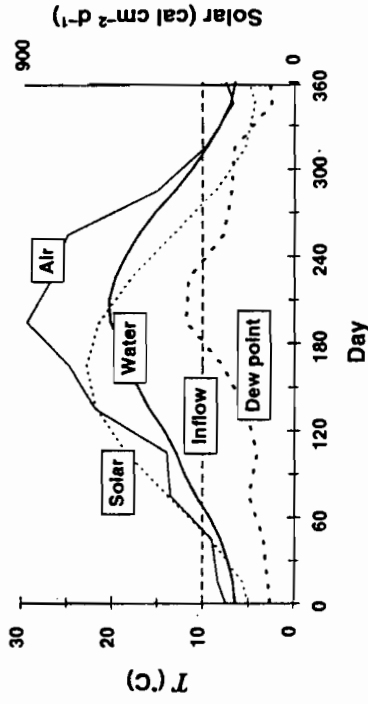


FIGURE E30.5-2
Simulation results.

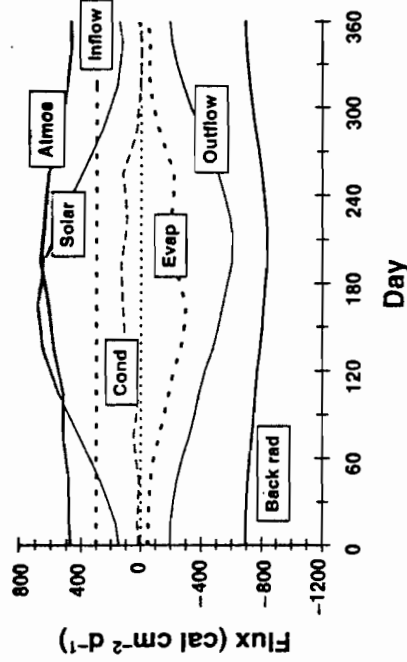


FIGURE E30.5-3
Components of the heat balance.

PROBLEMS

30.1. A $30 \times 10 \times 10$ cm brick is placed in a 2-m^3 container along with 1 m^3 of water. The container is perfectly insulated and is immediately sealed after the brick is added. If the brick, the water, and the air in the container are originally at $50, 10,$ and 20°C , respectively, determine the equilibrium temperature of the system. How much heat will be contained in the brick, the water, and the air before and after equilibrium? Express your results in kilocalories (kcal).

- 30.2. A heated discharge has a flow of $7 \text{ m}^3 \text{ s}^{-1}$ and a temperature of 40°C . A completely mixed cooling pond is to be built. If it is assumed that $J = -200 \text{ cal (cm}^2 \text{ d)}^{-1}$, determine the surface area so that the pond's exit temperature is 25°C .
- 30.3. Determine the net atmospheric surface heat flux so that the lake from Example 30.2 has a temperature of 30°C .
- 30.4. Calculate the air temperature if the dew-point temperature is 21°C and the relative humidity is 30%.
- 30.5 Repeat Example 30.4, but use a wind speed of 6 m s^{-1} .