LECTURE 31

Thermal Stratification

LECTURE OVERVIEW: I describe the vertical distribution of temperature in a lake. Then, I illustrate how vertical temperature distributions can be employed to quantify vertical transport in freshwater systems such as lakes.

A lake's vertical thermal regime has dual significance for the water-quality modeler. As mentioned throughout this text, temperature has a strong influence on the rates of chemical and biological reactions. However, it also has additional significance as a tracer of transport in the water column. In fact, heat balances are a primary tool for estimating vertical mixing rates in freshwater systems. Before describing how this is done, I will briefly describe the seasonal temperature changes in the water column of a lake.

31.1 THERMAL REGIMES IN TEMPERATE LAKES

Hutchinson (1957) defines temperate lakes as those "with temperature above 4°C in winter, thermal gradients large, two circulation periods in spring and late autumn." Although other lake types can be severely polluted,[†] the present discussion focuses on temperate lakes because many of the world's developed areas are in temperate

[†] For example lakes that never mix (*amictic*) or mix incompletely (*meromictic*) are extremely sensitive to pollutant inputs. In addition, because of growing urbanization and industrialization in the world's tropical regions, many nontemperate lakes are being subjected to severe water-quality stress. Although some of the approaches in this section might serve as a starting point for modeling these systems, additional research is needed for nontemperate lakes.

climates and, consequently, many lakes in these climates have been subject to pollution. Thus most engineering models have been developed for temperate systems.

The thermal regime of temperate lakes is primarily the result of the interplay of two processes: (1) heat and momentum transfer across the lake's surface and (2) the force of gravity acting on density differences within the lake. Depending on the season of the year, heat transfer tends to either raise or lower the temperature at the lake's surface as a consequence of a number of factors, including the magnitude of solar radiation, air temperature, relative humidity, wind speed, and cloud cover as described in the previous lecture. Winds blowing over the lake's surface tend to mix the surface waters and transfer heat and momentum down through the water column. The extent of this mixing is, in turn, inhibited by buoyancy effects. This relates to the fact that the density of water varies over the range of temperatures encountered in lakes (Fig. 31.1). Therefore, denser waters accumulate at the lake's bottom and are overlaid with lighter waters.

For example in Lake Ontario in August (Fig. 31.2), surface waters of 18°C with a density of 0.9986 g cm^{-3} overlay deep waters of 4°C with a density of 1.0000 g cm^{-3}. Although these density differences may seem small, considerable work must be expended to mix the entire column (that is, lift the heavier bottom waters against the force of gravity to mix them with the lighter surface waters). The energy to do this work comes from the wind. The result is that buoyancy works against and mitigates wind-induced turbulence. The interplay between these factors can be expressed quantitatively by a dimensionless parameter, the ***Richardson number,*** that represents the ratio of buoyancy to shear forces, as in

$$R_i = \frac{\text{buoyancy}}{\text{shear}} = \frac{(g/\rho)(\partial\rho/\partial z)}{(\partial u/\partial z)^2} \qquad (31.1)$$

where z = depth (L), which is positive in the downward direction
g = acceleration due to gravity (L T^{-2})
ρ = density of the fluid (M L^{-3})
$\partial\rho/\partial z$ = gradient of density with depth (M L^{-4})
$\partial u/\partial z$ = gradient of horizontal velocity with depth (T^{-1}) or shear

If R_i is significantly greater than a critical level (~ 0.25), a stable regime results. If R_i is significantly less than 0.25, connoting strong shear relative to stratification, then shear-induced turbulence is generated.

Figure 31.2 depicts the seasonal changes in the vertical temperature distribution of Lake Ontario. Although this lake is very deep, its thermal regime has many of the characteristic features of smaller temperate lakes. Some time after the disappearance of any ice cover in spring, temperatures throughout the water column rise to, or within a few degrees of, the maximum density of water (4°C). At this temperature, heating or cooling of the lake's surface results in very small density differences (Fig. 31.1*b*) and, consequently, only a small amount of wind stress is required to keep the water column well-mixed. In terms of the Richardson number, buoyancy forces (the numerator) are small, and therefore shear forces (the denominator) of small magnitude are sufficient to keep R_i below the critical level.

As spring progresses, solar radiation increases, air temperatures rise, and thermal stratification will be established in the near-surface waters. However, density

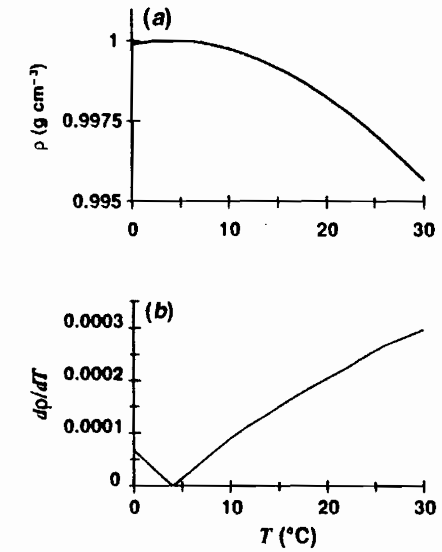

FIGURE 31.1
Plots of (*a*) density (g cm^{-3}) and (*b*) rate of change of density per rate of change of temperature (g cm^{-3} °C^{-1}) versus temperature (°C). Note that the maximum density of water occurs at 4°C.

gradients are neither large enough nor deep enough to prevent mixing of the water column by major storms. At the end of spring and the beginning of summer, surface heating increases to the point that mixing is confined to an upper layer. In terms of the bulk Richardson number, the fluid has reached the point where the density gradient is sharp enough that even large storms do not reduce R_i below the stability criterion. At this stage the lake is said to be stably stratified. The attainment of persistent stratification in a lake leads to the existence of three regimes: the upper (epilimnion) and lower (hypolimnion) layers separated by a narrow region of sharp temperature change—the ***thermocline*** or ***metalimnion.***[†]

During midsummer the net daily heat flux at the surface is low and, although the thermocline deepens gradually, the density gradient between the epi- and the hypolimnion remains strong and stable. Although transport of heat and energy across the thermocline occurs, it is at a low level and exchange between the upper and lower layers is at a minimum.

In late summer and fall, loss of heat due largely to falling air temperatures results in a net heat loss from the lake. As surface waters cool, they become more dense than underlying epilimnetic water. Since this is an unstable situation, strong vertical mixing called ***convection*** occurs. Together with increased winds during fall, the process erodes the metalimnion from above, giving the impression of a sinking thermocline. As the lake cools further, a point is reached at which the deepening surface layer becomes denser than the bottom layer, and complete mixing of the column occurs. This episode is called ***fall overturn.***

[†] The terms "thermocline" and "metalimnion" often are used interchangeably to designate the layer of sharp temperature change. Although the metalimnion is really a layer, the thermocline is actually the plane passing through the point of maximum decrease in temperature with depth.

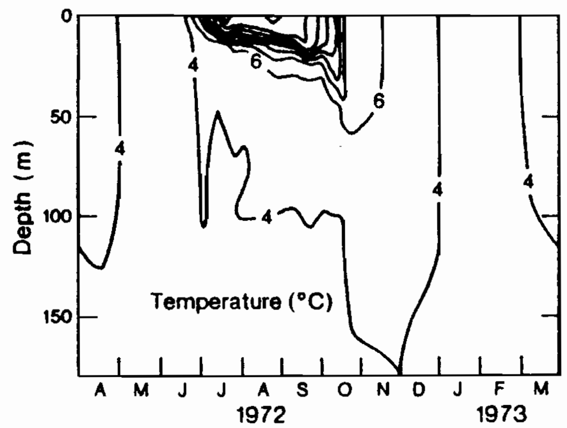

FIGURE 31.2
Depth-time diagram of isotherms (°C) at a mid-lake station in Lake Ontario, 1972– 1973.

The lake continues to be well-mixed and to lose heat as temperatures drop in winter. In some cases the surface water cools below 4°C and an inverse stratification results owing to the low density of water below 4°C (Fig. 31.1*a*). This inverse stratification is strengthened if ice forms at the lake's surface.

In summary the seasonal changes in a temperate lake can be idealized in both time and space. Temporally the cycle consists of two stages: a summer period of strong stratification and a nonstratified period of intense vertical mixing. Spatially the summer stratified period can be treated as consisting of two layers separated by an interface of minimal vertical mixing. These idealizations form the basis of some efforts to develop engineering models of vertical heat and mass distribution in temperate lakes.

31.2 ESTIMATION OF VERTICAL TRANSPORT

In previous lectures (for example Lecs. 8 and 15) we showed how gradients of conservative substances, such as chloride, could be used to estimate horizontal diffusion. Because of the large seasonal temperature gradients in temperate lakes, heat balances serve an analogous role in vertical models. This implies that water motion has an identical effect on determining heat and mass transport in the water column. Although this analogy does not hold strictly for large particles or dense solutions, it is a good first approximation for most pollutants.

We can now write a heat budget in a similar fashion to those developed in the previous lecture. Consider the simplest case of a temperate lake during the mid-summer stratified period. At this time (Fig. 31.2) the temperatures in the epi- and hypolimnion are fairly uniform, with the major gradient occurring at the thermocline. Although the thermocline deepens during this period, we assume that the descent is, at most, very gradual. A simple model for this case consists of two well-mixed layers of constant thickness separated by an interface across which diffusive transport occurs (Fig. 31.3). Therefore the metalimnion is not modeled explicitly. Rather the thermocline acts as the interface between the surface and the bottom layers. For such a system, heat balances can be written for each of the layers as

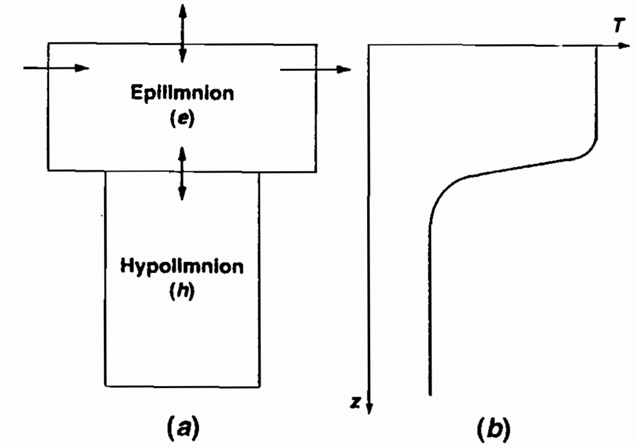

FIGURE 31.3
Idealization of summer stratified period in a temperate lake. (*a*) The segmentation of the water column into a well-mixed epilimnion and hypolimnion corresponding to the vertical temperature distribution in (*b*).

$$V_e \rho C_p \frac{dT_e}{dt} = Q\rho C_p T_{in}(t) - Q\rho C_p T_e \pm J A_s + v_t A_t \rho C_p (T_h - T_e) \quad (31.2)$$

$$V_h \rho C_p \frac{dT_h}{dt} = v_t A_t \rho C_p (T_e - T_h) \quad (31.3)$$

where the subscripts e and h designate the epilimnion and hypolimnion, respectively, T = temperature (°C), ρ = density (g cm^{-3}), C_p = specific heat (cal g^{-1} °C^{-1}), Q = volumetric flow rate of all water sources entering the system (g cm^{-3}), $T_{in}(t)$ = average inflow temperature of these sources (°C), A_s = lake's surface area (cm^2), J = surface heat flux (cal cm^{-2} d^{-1}), v_t = thermocline heat transfer coefficient (cm d^{-1}), and A_t = thermocline area (cm^2).

It should be noted that exchange across the thermocline also can be parameterized as a vertical diffusion coefficient E_t, where E_t has units of cm^2 d^{-1} and is related to the heat exchange coefficient by

$$E_t = v_t H_t \quad (31.4)$$

where H_t = thermocline thickness (cm). This alternative expression is useful in comparing the magnitude of vertical mixing with other turbulent processes. However, it has the disadvantage that it requires specification of the thermocline thickness to parameterize vertical mixing.

Equation 31.3 can now be used to estimate the thermocline heat exchange coefficient. To do this we assume that during the summer stratified period, the epilimnion temperature is constant (see Fig. 31.4). If this is true, Eq. 31.3 can be written as

$$\frac{dT_h}{dt} + \lambda_h T_h = \lambda_h \overline{T}_e \quad (31.5)$$

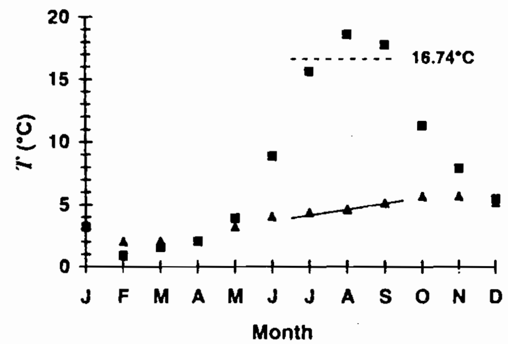

FIGURE 31.4
Plot of mean monthly epilimnetic and hypolimnetic temperatures for Lake Ontario during 1967 from Thomann and Segna (1980). The mean epilimnion temperature for the summer stratified period is displayed (dashed line) along with the simulated hypolimnetic temperature (solid line).

where $\overline{T}_e$ designates that the epilimnion temperature is constant and λ_h is the eigenvalue for the hypolimnion,

$$\lambda_h = \frac{v_t A_t}{V_h} \tag{31.6}$$

Equation 31.5 is identical in form to the completely mixed lake model, with step input described previously in Lec. 4. If the hypolimnion temperature at the beginning of the summer period is $T_{h,i}$, Eq. 31.5 can be solved for

$$T_h = T_{h,i}e^{-\lambda_h t} + \overline{T}_e(1 - e^{-\lambda_h t}) \tag{31.7}$$

Equation 31.7 can then be arranged to estimate the heat exchange coefficient across the thermocline (Chapra 1980),

$$v_t = \frac{V_h}{A_t t_s} \ln\left(\frac{\overline{T}_e - T_{h,i}}{\overline{T}_e - T_{h,s}}\right) \tag{31.8}$$

where t_s = time after the onset of stratification at which the hypolimnion temperature $T_{h,s}$ is measured.

EXAMPLE 31.1. THERMOCLINE TRANSFER COEFFICIENT. Lake Ontario is strongly stratified during the months of July, August, and September. During this time the thermocline is at a depth of approximately 15 m. The average epilimnetic temperature over the period is approximately 16.74°C and the surface area of the thermocline is approximately $15{,}000 \times 10^6$ m^2. The temperature in the hypolimnion (volume = 1380×10^9 m^3) rises from 4.20°C in the beginning of July to 5.38°C at the end of September (Table 31.1). Use Eq. 31.8 to estimate the thermocline transfer coefficient. In addition compute the thermocline diffusion coefficient if H_t = 7 m.

Solution: Equation 31.8 can be used to compute

$$v_t = \frac{1380 \times 10^{15}\text{ cm}^3}{15{,}000 \times 10^{10}\text{ cm}^2(90\text{ d})} \ln\left(\frac{16.74 - 4.20}{16.74 - 5.38}\right) = 10.1\text{ cm d}^{-1}$$

which in turn can be used to compute a diffusion coefficient

$$E_t = 10.1\text{ cm d}^{-1}(700\text{ cm})\left(\frac{\text{d}}{86,400\text{ s}}\right) = 0.082\text{ cm}^2\text{ s}^{-1}$$

TABLE 31.1
Mean monthly epilimnetic (0 to 15 m) and hypolimnetic (15 m to bottom) temperatures for Lake Ontario (Chapra and Reckhow 1983).

Month	T_e (°C)	T_h (°C)	Month	T_e (°C)	T_h (°C)	Month	T_e (°C)	T_h (°C)
Dec	5.47	5.18	May	3.91	3.20	Oct	11.29	5.64
Jan	3.18	3.30	Jun	8.86	4.05	Nov	7.92	5.68
Feb	0.89	2.00	Jul	15.58	4.35	Dec	5.47	5.18
Mar	1.55	2.04	Aug	18.56	4.61	Jan	3.18	3.30
Apr	2.06	2.04	Sep	17.74	5.12			

These coefficients can now be employed in conjunction with Eq. 31.7 to calculate the hypolimnetic temperature and response time. First, the eigenvalue can be computed as (Eq. 31.6)

$$\lambda_h = \frac{10.1(15{,}000 \times 10^{10})}{1380 \times 10^{15}} = 0.001087\text{ d}^{-1}\ (= 0.397\text{ yr}^{-1})$$

Then Eq. 31.7 can be used to calculate

$$T_h = 4.2e^{-0.001087t} + 16.75(1 - e^{-0.001087t})$$

Values calculated with this formula are plotted in Fig. 31.4. As can be seen, the fit seems linear because the response time of the hypolimnion is so long,

$$t_{95} = \frac{3}{\lambda_h} = \frac{3}{0.397} = 7.6\text{ yr}$$

The transfer coefficient can also be used to compute mass transfer of substances across the thermocline. For example in Lake Ontario during the summer, the concentrations of soluble reactive phosphorus in the epilimnion and hypolimnion are approximately 3.1 and 8.6 μg L^{-1}, respectively. The mass-transfer coefficient from the hypolimnion to the epilimnion can be estimated as

$$10.1\text{ cm d}^{-1}(15{,}000 \times 10^{10}\text{ cm}^2)(8.6 - 3.1)\text{ mg m}^{-3}\left(\frac{\text{m}^3}{10^6\text{ cm}^3}\right)\left(\frac{365\text{ d}}{\text{yr}}\right)\left(\frac{\text{mta}}{10^9\text{ mg yr}^{-1}}\right)$$
$$= 3001\text{ tonnes yr}^{-1}$$

This estimate can be put into perspective by comparing it to the 12,000 metric tons per year (mta) of total phosphorus that enters the lake from external sources. The fact that diffusive transport to the epilimnion across the thermocline amounts to about 25% of the external loading suggests its relative importance in regulating surface production of phytoplankton during the growing season.

Snodgrass (1974) has summarized estimates of the thermocline diffusion coefficient E_t for a number of lakes. The values are positively correlated with mean depth (Fig. 31.5) and range over several orders of magnitude from 0.003 to 2.4 cm^2 s^{-1}. Note that the following formula can be derived from the plot:

$$E_t = 7.07 \times 10^{-4} H^{1.1505} \tag{31.9}$$

where E_t is in cm^2 s^{-1} and H = mean depth (m). With knowledge of the

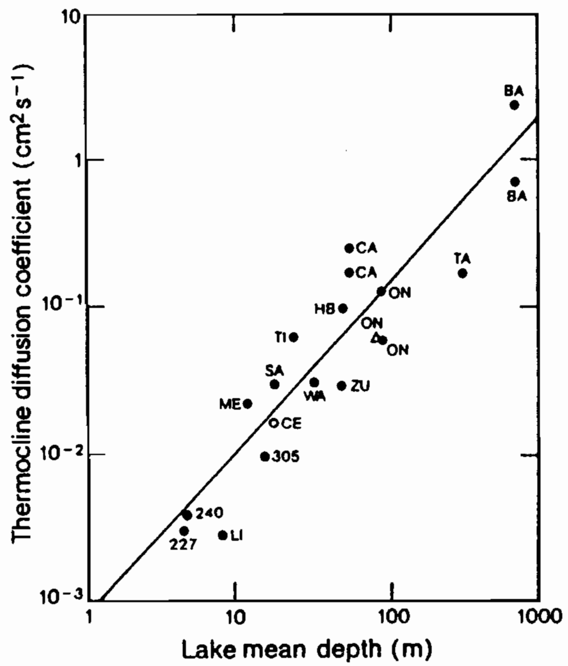

FIGURE 31.5
Thermocline diffusion coefficient ($cm^2\ s^{-1}$) versus lake mean depth for a number of temperate lakes. Redrawn from Snodgrass (1974) with new data from Lake Ontario (Δ) and the central basin of Lake Erie (O).

approximate thickness of the metalimnion, the plot or the equation can be used to make a first estimate of the thermocline exchange coefficient for a lake.

Now that we have estimated the thermocline exchange coefficient, it is relatively straightforward to combine the two-layer lake model with the surface heat exchange model described in the previous lecture.

EXAMPLE 31.2. TIME-VARIABLE HEAT BALANCE FOR A STRATIFIED LAKE. Compute the annual heat budget for the same pond used in Examples 30.4 and 30.5. Note that the following additional information is available:

$$V_e = 175{,}000\ m^3 \qquad A_t = 11{,}000\ m^2 \qquad A_s = 25{,}000\ m^2$$
$$V_h = 75{,}000\ m^3 \qquad H_t = 3\ m$$

Assume that all atmospheric interactions are limited to the epilimnion and use Eq. 31.9 to determine the thermocline transfer coefficient for the summer stratified period (April 1 through September 30). Also assume that the lake does not stratify during the winter.

Solution: First, we must determine the thermocline transfer coefficient. To do this the pond's mean depth can be computed as

$$H = \frac{V}{A_s} = \frac{250{,}000}{25{,}000} = 10\ m$$

Then Eq. 31.9 can be used to compute

$$E_t = 7.07 \times 10^{-4}(10)^{1.1505} = 0.01\ cm^2\ s^{-1}$$

The transfer coefficient can be determined as

$$v_t = \frac{E_t}{H_t} = \frac{0.01\ cm^2\ s^{-1}}{3\ m}\left(\frac{m^2}{10{,}000\ cm^2}\right)\left(\frac{86{,}400\ s}{d}\right) = 0.0288\ m\ d^{-1}$$

For the unstratified period, a large enough coefficient is used to simulate complete mixing. The magnitude of this exchange coefficient is estimated by trial and error.

Equations 31.2 and 31.3 can now be integrated with a method such as the fourth-order Runge-Kutta approach described in Lec. 7. The results are displayed in Fig. E31.2. We have included the single-layer model from Example 30.5 for comparison. Observe how the single-layer-model temperature falls between the results for the two-layer case. As expected, because it is subject to meteorological influences, the epilimnion temperature is warmer during the stratified period.

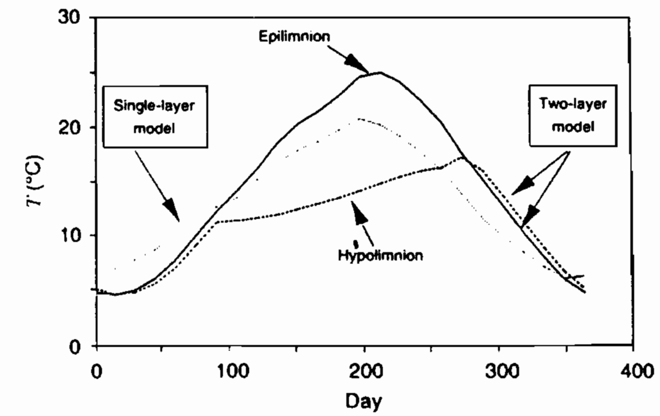

FIGURE E31.2

31.3 MULTILAYER HEAT BALANCES (ADVANCED TOPIC)

Although the lumped approach of the previous section is widely used, one-dimensional distributed approaches are also employed. The primary reason for using such approaches relates to the fact that the hypolimnion is not completely mixed. In fact it is more accurately characterized as a one-dimensional system where heat transport occurs by diffusion.

Such a distributed approach may be less important for unproductive or very deep systems (Fig. 31.6*a*). For these cases, vertical gradients of constituents may not be significant in the hypolimnion. Thus a well-mixed bottom layer serves as an adequate approximation.

However, for shallower, productive systems, significant hypolimnetic gradients can occur during stratification (Fig. 31.6*b*). In most cases such gradients are created by sediment release of constituents. Because the hypolimnion is governed by diffusion, concentrations rise because of the sediment sources. Thus, higher concentrations first develop near the sediment-water interface and slowly diffuse up through the water column. A distributed characterization is required to simulate such gradients and the associated mass transport.

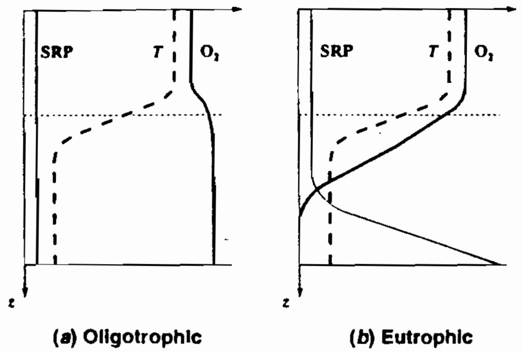

FIGURE 31.6
Typical vertical profiles of temperature, oxygen, and soluble reactive phosphorus in (*a*) an oligotrophic and (*b*) a eutrophic lake. For the oligotrophic system, sediment sources are not strong enough to induce vertical gradients in the bottom water. In contrast eutrophic systems typically have productive sediment that take up oxygen and release nutrients. Consequently, strong vertical gradients usually occur.

Two approaches have been developed to model vertical temperature distributions. The first is based on a ***turbulent diffusion approach*** (Fig. 31.7*a*). Numerical solutions are developed by dividing the vertical dimension into segments. Heat is supplied to the surface and then distributed through depth by diffusion in a fashion similar to the distributed models described earlier in the book (for example the elongated reactors described in Part II).

The second method, called a ***mixed-layer approach*** (Fig. 31.7*b*), uses a mechanical energy balance to predict the thickness of the epilimnion. This thick surface layer is then modeled as a well-mixed segment. The hypolimnion is modeled as a series of layers using the turbulent diffusion approach.

Because of its simplicity we focus on the turbulent diffusion approach. A one-dimensional heat balance can be written for the water column as

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left[E(z)\frac{\partial T}{\partial z}\right] + \frac{J_{sn}(z)A_s}{\rho C_p} \tag{31.10}$$

where T = temperature
z = depth, measured downward from the surface
$E(z)$ = a vertical turbulent diffusion coefficient
$J_{sn}(z)$ = net solar radiation distributed by depth
A_s = surface area
ρ = water density
C_p = water specific heat

Note that the solar radiation is distributed by depth according to the formula

$$J_{sn}(z) = (1 - F_a)J_{sn}e^{-k_e z} \tag{31.11}$$

where F_a = fraction of the solar radiation that is absorbed immediately at the surface
J_{sn} = net solar radiation delivered to the surface
k_e = extinction coefficient

The following boundary conditions are required at the water surface and the bottom:

$$-E(0)\frac{\partial T}{\partial z} = \frac{J_{sn} - (1 - F_a)J_{sn}}{\rho C_p} \tag{31.12}$$

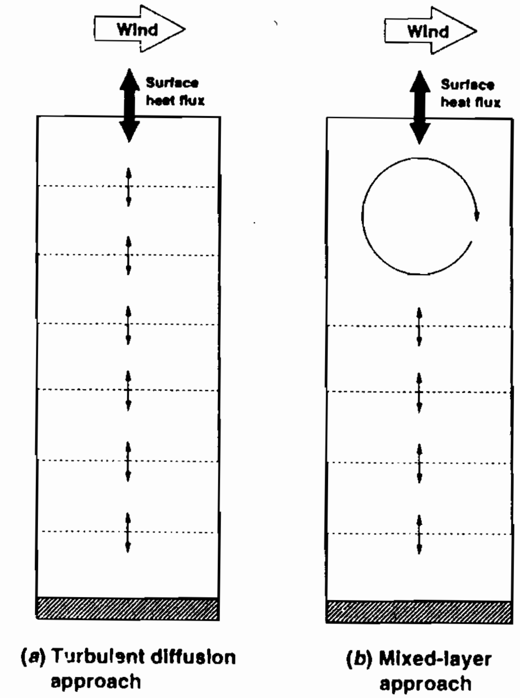

FIGURE 31.7
Contrast between the two primary models used to distribute heat vertically in lakes and reservoirs.

and

$$\frac{\partial T}{\partial z}(H) = 0 \tag{31.13}$$

The first condition specifies the heat flux at the air-water interface. The second condition sets an insulated condition at the bottom.

Also observe how a portion of the solar radiation is absorbed within a few centimeters of the surface. The remainder is absorbed in an exponential fashion as it radiates down through the water column.

At this point the model formulated above merely distributes heat through the water column by mixing. The key to simulating stratification relates to how the vertical diffusion coefficient varies with depth. A variety of approaches have been proposed. One of the first was that of Munk and Anderson (1948). This model assumed that the diffusion coefficient was a function of the Richardson number,

$$E(z) = \frac{E_0}{(1 + aR_i)^{3/2}} \tag{31.14}$$

where E_0 = the diffusion coefficient at neutral stability
R_i = Richardson number
a = a constant

A variety of formulations have been proposed to relate E_0 to depth and wind velocity. We will use the simplest formulation,

$$E_0 = c\omega_\theta \tag{31.15}$$

where c = an empirical constant and ω_θ = wind shear velocity,

$$\omega_\theta = \sqrt{\frac{\tau_s}{\rho_w}} \tag{31.16}$$

in which ρ_w = water density and τ_s = shear stress at the air-water interface,

$$\tau_s = \rho_{air} C_d U_w^2 \tag{31.17}$$

where ρ_{air} = air density
C_d = a drag coefficient = $0.00052 U_w^{0.44}$
U_w = wind speed

Aside from Eq. 31.1, alternative formulations are available to compute the Richardson number. For example

$$R_i = \frac{-(g/p)(\partial \rho/\partial x)}{\omega_0/(z_s - z)^2} \tag{3.18}$$

where z_s = the water surface elevation (m).

Although the foregoing sequence of formulas may seem involved, their ultimate effect is embodied in Eq. 31.14. In essence the numerator is a manifestation of the kinetic energy delivered to the surface by the wind. Thus, as we would expect, the vertical diffusion within the lake will increase for higher winds. The denominator decreases the diffusion as the Richardson number increases. Thus, as the density gradient increases or as the velocity gradient decreases, the diffusion coefficient decreases and stratification can occur.

The foregoing is an introduction to vertical temperature modeling in lakes. It reviews one of many formulations that have been proposed to model this phenomenon. Ford and Johnson (1986) and Henderson-Sellers (1984) can be consulted for additional information.

PROBLEMS

31.1. A pond has the following characteristics:

Epilimnion volume = 150,000 m^3 | Thermocline area = 10,000 m^2
Hypolimnion volume = 50,000 m^3 | Inflow = outflow = 5000 m^3 d^{-1}
Surface area = 25,000 m^2 | Thermocline thickness = 3 m

The pond's inflow has a temperature of 10°C and enters and leaves the hypolimnion. In addition it is subject to the following meteorological conditions:

Net solar radiation = 250 cal cm^{-2} d^{-1} | Dew-point temperature = 15°C
Air temperature = 20°C | Wind speed = 2 m s^{-1}

Calculate the steady-state temperatures of the epilimnion and hypolimnion.

31.2. The Central Basin of Lake Erie had the following general characteristics for the summer stratified period from 1967 through 1972:

Mean depth = 17.8 m | Hypolimnion thickness = 4 m
Volume hypolimnion = 40 km^3 | Epilimnetic temperature = 19.94°C
Area thermocline = 10,000 km^2 | Epilimnion oxygen = 9.66 mg L^{-1}
Metalimnion thickness = 7 m | Total P concentration = 20 μg L^{-1}

In addition, the following data can be extracted from temperature and oxygen time series:

	June 1	September 2
Hypolimnion temperature (°C)	7.77	12.5
Hypolimnion oxygen (mg L^{-1})	10.72	0.60

(*a*) Determine the thermocline diffusion coefficient in cm^2 s^{-1} and compare it with the value from the Snodgrass plot.
(*b*) Determine:
(*i*) The apparent areal hypolimnetic oxygen demand in g m^{-2} d^{-1}
(*ii*) The actual areal hypolimnetic oxygen demand in g m^{-2} d^{-1} corrected for diffusion
(*iii*) The empirical areal hypolimnetic oxygen demand in g m^{-2} d^{-1} as a function of total P concentration (Eq. 29.30)
(*c*) Compare and discuss the results of (*b*).

31.3 Use the fourth-order RK method to compute the hypolimnetic temperature for a lake having the characteristics given in Table P31.3. Use a value of 0.13 cm^2 s^{-1} for the thermocline diffusion coefficient for the summer stratified period. Also assume that the lake does not stratify during the winter. Use the same epilimnion temperatures as in Table 31.1.

TABLE P31.3
Lake characteristics

Parameter	Value	Units
Lake surface area	25,000	m^2
Thermocline area	11,000	m^2
Epilimnion volume	175,000	m^3
Hypolimnion volume	75,000	m^3
Thermocline thickness	3	m
Epilimnion thickness	7	m
Hypolimnion thickness	6.8	m
Inflow = outflow	274 × 10^4	m^3 yr^{-1}
Start of summer stratification	120	d
End of stratification	290	d