Empirical Relations for Longitudinal Dispersion in Streams

By Prabhata K. Swamee,[1] Santosh K. Pathak,[2] and Mohammad Sohrab[3]

ABSTRACT: Although several methods are available for dispersion in natural streams, no method is accurate enough to satisfactorily predict the time variation of stream pollution concentration. Further, limited studies exist for dispersion of nonconservative pollutants. In this paper a six-parameter concentration equation for dispersion of conservative and nonconservative pollutants has been proposed. The parameters of the equation have been related to hydraulic variables and stream geometry. Using these predictors, the equation is fairly accurate for concentration predictions. It is hoped that the equation is useful in water quality management studies.

INTRODUCTION

When a mass of pollutant is injected instantaneously in a stream, it mixes in vertical and transverse directions leading to a nearly uniform concentration over the cross section. Subsequently, because of flow characteristics and stream properties, this uniform concentration disperses in the longitudinal direction. If the pollutant is nonconservative, it decays with time due to biodegradation and adsorption, etc.

Presented herein is a concentration equation for pollutants. The parameters involved in the equation have been related to the flow properties and the channel geometry.

PREVIOUS WORK

Conservative Pollutants

Taylor (1954) gave the following equation for the average concentration of a conservative pollutant at a section:

$$\frac{\partial c}{\partial t} + V \frac{\partial c}{\partial x} = D_L \frac{\partial^2 c}{\partial x^2} \tag{1}$$

where c = average concentration; t = time; V = average velocity; x = longitudinal distance; and D_L = longitudinal dispersion coefficient. The well-known solution of (1) for instantaneous injection of mass M_o at $t = 0$ is (Sayre 1968)

$$c = \frac{M_o}{A\sqrt{4\pi D_L t}} \exp\left[\frac{-(x - Vt)^2}{4 D_L t}\right] \tag{2}$$

where A = flow area. Many researchers have developed the predictors for D_L (Thackston and Krenkel 1967; Fischer 1975; McQuivey and Keefer 1975). Liu (1977) stated that these predictors yield D_L varying from 300 to 4,000 times smaller than the measured values in the Missouri river. However, the predictors for D_L proposed by Liu (1977) and Seo and Cheong (1998) show good performance in natural streams.

Nordin and Sabol (1974) analyzed longitudinal dispersion data from 51 field experiments in American rivers and found that only six cases were in accordance with (2). Similarly, Day (1975) analyzed data of 49 experiments on mountain streams in New Zealand and concluded that the process of dispersion in natural streams did not follow (2). Day and Wood (1976) have shown that the dispersion is a self-similar process for which the concentration versus time curve can be plotted in the form of similarity curves. Beltaos (1980) proposed a sim-

ilarity model involving two dispersion parameters. Recently Singh et al. (1992) analyzed various concentration time curves observed in laboratory and field channels by other investigators and obtained modified similarity predictors. However, these approaches do not conserve the pollutant mass.

Nonconservative Pollutants

For nonconservative pollutants (2) is invalid because of pollutant loss due to biodegradation and adsorption, etc. Considering loss for the detention of pollutants in lateral and vertical dead zones, a modified solution of (1) for nonconservative pollutants was given as (Bansal 1971)

$$c = \frac{M_o}{AD_R\sqrt{4\pi D_L t}} \exp\left[-\frac{(x - Vt)^2}{4\pi D_L}\right]$$

$$\cdot \left[1 - \frac{xT}{16 D_L t} - \frac{x}{4Vt} \frac{\ln\left(\frac{Q}{160 D_L T}\right)}{\ln\left(\frac{xV}{102,400 D_L}\right)} - \frac{K_d x}{4Vt}\right] \tag{3}$$

where D_R = regional dispersion factor, which varies from stream to stream; Q = discharge; T = flow width; and K_d = decay constant. Bansal (1971) took K_d in (3) as 0.5 for all streams, whereas D_R was obtained by matching the computed and observed concentrations. O'Loughlin and Bowmer (1975) stated that the concentration of pollutants in the dispersion zone is governed by the convective diffusion equation

$$\frac{\partial c}{\partial t} + V \frac{\partial c}{\partial x} = D_L \frac{\partial^2 c}{\partial x^2} - Kc \tag{4}$$

where K = decay constant. The solution of (4) for instantaneous injection of mass at $t = 0$, as given by Carlsaw and Jaeger (1959), is

$$c = \frac{M_o}{A\sqrt{4\pi D_L t}} \exp\left[\frac{-(x - Vt)^2}{4 D_L t} - Kt\right] \tag{5}$$

O'Loughlin and Bowmer (1975) proposed an iterative method for determination of D_L. For the determination of K, dispersion data of both conservative and nonconservative pollutants are required.

The limitation of Fickian models (1) and (4) is that they are not able to accurately represent highly skewed concentration profiles observed in natural streams.

CONCENTRATION EQUATION

The cumulative mass M passing through a monitoring station up to time t is given by

$$M = \int_0^t cQ \, dt \tag{6}$$

[1]Prof., Civ. Engrg. Dept., Univ. of Roorkee, Roorkee 247667 India.

[2]Prof., Civ. Engrg. Dept., Univ. of Roorkee, Roorkee, India.

[3]Res. Fellow, Civ. Engrg. Dept., Univ. of Roorkee, Roorkee, India.

At a monitoring station c remains zero until the inception time t_x, at which the pollutant first reaches the station. Subsequently, the M-t curve first rises rapidly and thus saturates to M_x given by

$$M_s = \int_0^\infty cQ \, dt \tag{7}$$

Plotting the fast-rising portion of M-t curve on a double logarithmic paper indicated the following equation:

$$M = M_o \left(\frac{t - t_x}{t_* - t_x} \right)^m \quad t \geq t_x \tag{8}$$

where m = slope of the rising limb; and t_* = scaling parameter [Fig. 1(b)].

On account of the decay of nonconservative pollutant, M_x progressively reduces with the distance x. Considering exponential decay of mass, M_x was expressed as

$$M_x = M_o \exp(-t_x/t_d) \tag{9}$$

where t_d = decay time. For a conservative pollutant the decay time is infinity. The inception time t_x occurring in (9) is a direct function of x. Thus, M_x decreases with x. Combining (8) and (9) through a transition exponent n, the following equation was obtained:

$$M = M_o \left[\left(\frac{t_* - t_x}{t - t_x} \right)^{m/n} + \exp \left(\frac{t_x}{nt_d} \right) \right]^{-n}; \quad t \geq t_x \tag{10}$$

Differentiating (10) with respect to t and putting $dM/dt = cQ$, the following equation for pollutant concentration was obtained:

$$c = \frac{mM_o}{(t_* - t_x)Q} \left[\left(\frac{t_* - t_x}{t - t_x} \right)^{(m-1)/(n+1)} \right. $$
$$\left. + \left(\frac{t - t_x}{t_* - t_x} \right)^{(m+n)n/(n+1)} \exp \left(\frac{t_x}{nt_d} \right) \right]^{-(n+1)} \tag{11}$$

Differentiating (11) with respect to t, equating it to zero and simplifying, the peak concentration time t_p was obtained as

$$t_p = t_x + (t_* - t_x) \left[\frac{n(m - 1)}{m + n} \right]^{n/m} \exp \left(-\frac{t_x}{mt_d} \right) \tag{12}$$
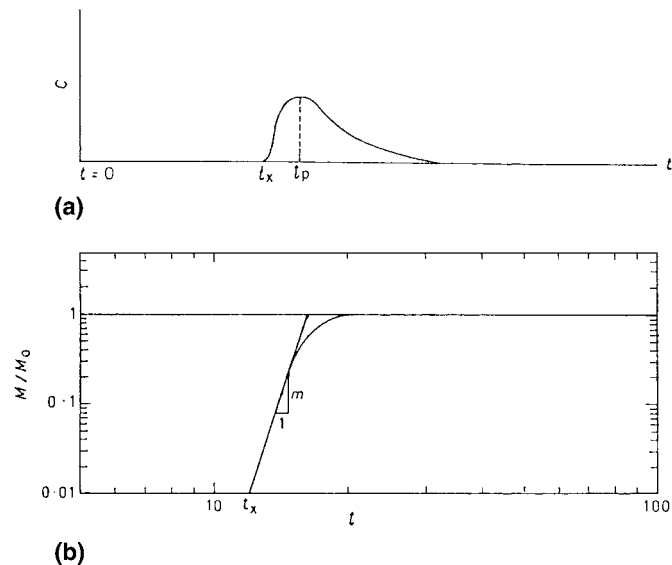

(a)

(b)

FIG. 1. Definition Sketch: (a) Concentration Curve; (b) Cumulative Mass Curve

For t_*, (12) was rewritten as

$$t_* = t_x + (t_p - t_x) \left[\frac{m + n}{n(m - 1)} \right]^{n/m} \exp \left(\frac{t_x}{mt_d} \right) \tag{13}$$

Eliminating t_* between (10) and (13), M was found as

$$M = M_o \exp \left(-\frac{t_x}{t_d} \right) \left[\frac{m + n}{n(m - 1)} \left(\frac{t_p - t_x}{t - t_x} \right)^{m/n} + 1 \right]^{-n} \tag{14a}$$

Similarly, eliminating t_* between (11) and (13), c was found as

$$c = \frac{mM_o \exp(-t_x/t_d)}{(t_p - t_x)Q} \left(\frac{t - t_x}{t_p - t_x} \right)^{m-1} \left\{ \left[\frac{m + n}{n(m - 1)} \right]^{n/(n+1)} \right.$$
$$\left. + \left[\frac{n(m - 1)}{m + n} \right]^{1/(n+1)} \left(\frac{t - t_x}{t_p - t_x} \right)^{m/n} \right\}^{-(n+1)} \tag{14b}$$

Putting $t = t_p$ in (14b), the peak concentration c_p was obtained as

$$c_p = \frac{(m + n)M_o}{(n + 1)(t_p - t_x)Q} \left[\frac{n(m - 1)}{m(n + 1)} \right]^n \exp(-t_x/t_d) \tag{15a}$$

Further, by putting $t = t_p$ in (14a) and using (15a), the mass M_p corresponding to the peak concentration was

$$M_p = \frac{(n + 1)(t_p - t_x)Qc_p}{m + n} \tag{15b}$$

Using (15a), the concentration equation (14b) was simplified to

$$c = c_p \left(\frac{t - t_x}{t_p - t_x} \right)^{m-1} \left[\frac{m + n}{m(n + 1)} + \frac{n(m - 1)}{m(n + 1)} \left(\frac{t - t_x}{t_p - t_x} \right)^{m/n} \right]^{-(n+1)} \tag{16}$$

The parameters m, n, t_x, t_p, and t_d may be called dispersion parameters. Plotting the concentration versus $t - t_x$ curve (Fig. 2), it is possible to evaluate the dispersion parameters. The inception time t_x, peak concentration c_p, and the time of the peak concentration t_p can be readily determined from a c-t curve [Fig. 1(a)]. As the slope of the rising limb of the double logarithmic plot of c versus $(t - t_x)$ curve is $m - 1$, the pa-
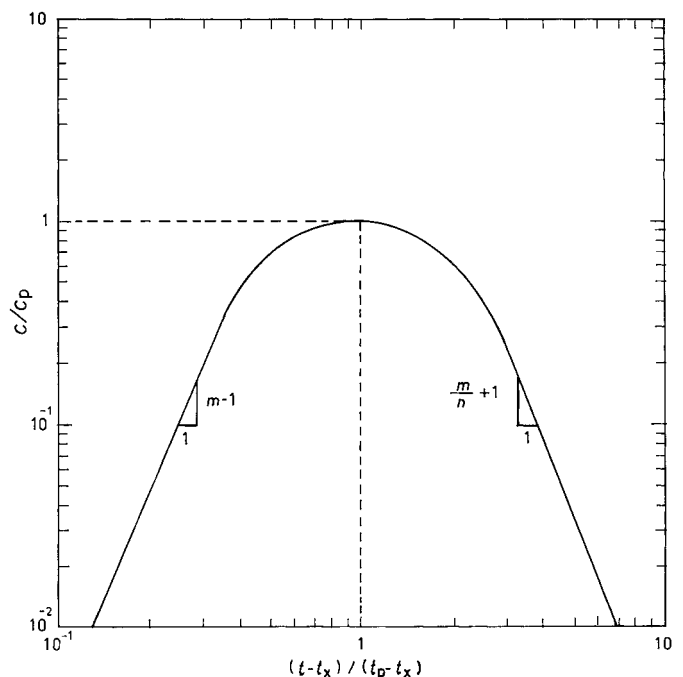

FIG. 2. Definition Sketch of m and n

rameter m is determined. Similarly, knowing m, the corresponding slope of the recession limb $-(m/n + 1)$ determines the exponent n.

Evaluation of Dispersion Parameters

The dispersion parameters can be obtained by minimizing the average error between the observed concentration profile and the profile represented by (16). For this, assuming arbitrary dispersion parameters, the concentration c_{cj} at time t_j was obtained by (16). Comparing c_{cj} with the observed concentration c_{oj}, the average error E was obtained as

$$E = \frac{100}{N} \sum_{j=1}^{N} \frac{|c_{cj} - c_{oj}|}{c_{oj}} \qquad (17)$$

where N = number of concentration observations. E is a function of the dispersion parameters. Minimizing E, the dispersion parameters were obtained.

FUNCTIONAL DEPENDENCE OF DISPERSION PARAMETERS

The dispersion parameters depend on the flow properties and the channel geometry. Adopting hydraulic radius R and the gravitational acceleration g as the repeating variables, the following nondimensional groups were formed:

- Distance from the injection point x: x/R
- Flow area A: A/R^2
- Flow velocity V: $V/\sqrt{gR}$
- Time t: $t\sqrt{g/R}$

Using these nondimensional groups, the dispersion parameters were expressed as

$$m = k_0 + k_1 \left(\frac{x}{R}\right)^{k_2} \left(\frac{A}{R^2}\right)^{k_3} \left(\frac{V}{\sqrt{gR}}\right)^{k_4} \qquad (18a)$$

$$n = k_5 \left[1 + \left(\frac{x}{R}\right)^{k_6} \left(\frac{A}{R^2}\right)^{k_7} \left(\frac{V}{\sqrt{gR}}\right)^{k_8} \right]^{-1} \qquad (18b)$$

$$t_x = k_9 \left(\frac{x}{R}\right)^{k_{10}} \left(\frac{A}{R^2}\right)^{k_{11}} \left(\frac{V}{\sqrt{gR}}\right)^{k_{12}} \sqrt{\frac{R}{g}} \qquad (18c)$$

$$t_p = k_{13} \left(\frac{x}{R}\right)^{k_{14}} \left(\frac{A}{R^2}\right)^{k_{15}} \left(\frac{V}{\sqrt{gR}}\right)^{k_{16}} \sqrt{\frac{R}{g}} \qquad (18d)$$

$$t_d = k_{17} \left(\frac{x}{R}\right)^{k_{18}} \left(\frac{A}{R^2}\right)^{k_{19}} \left(\frac{V}{\sqrt{gR}}\right)^{k_{20}} \sqrt{\frac{R}{g}} \qquad (18e)$$

where $k_0 - k_{20}$ = constants to be determined experimentally.

Evaluation of Constants

For the ith concentration profile, the dispersion parameters m_{oi}, n_{oi}, t_{xoi}, t_{poi}, and t_{doi} were obtained by minimizing (17). Similarly, using arbitrary values of constants $k_0 - k_{20}$ in (18a)–(18e), the dispersion parameters m_{ci}, n_{ci}, t_{xci}, t_{pci}, and t_{dci} were obtained for the ith concentration profile. Comparing these two sets of dispersion parameters, the corresponding average percentage errors E_m, E_n, E_{tx}, E_{tp}, and E_{td} were expressed as

$$E_m = \frac{100}{N} \sum_{i=1}^{N} \frac{|m_{ci} - m_{oi}|}{m_{oi}} \qquad (19a)$$

$$E_n = \frac{100}{N} \sum_{i=1}^{N} \frac{|n_{ci} - n_{oi}|}{n_{oi}} \qquad (19b)$$

$$E_{tx} = \frac{100}{N} \sum_{i=1}^{N} \frac{|t_{xci} - t_{xoi}|}{t_{xoi}} \qquad (19c)$$

$$E_{tp} = \frac{100}{N} \sum_{i=1}^{N} \frac{|t_{pci} - t_{poi}|}{t_{poi}} \qquad (19d)$$

$$E_{td} = \frac{100}{N} \sum_{i=1}^{N} \frac{|t_{dci} - t_{doi}|}{t_{doi}} \qquad (19e)$$

where N = number of concentration profiles. Minimizing E_m, E_n, E_{tx}, E_{tp}, and E_{td} by varying $k_0 - k_{20}$, the optimal constants were determined.

DESCRIPTION OF DATA

For evaluation of the constants, the data of Nordin and Sabol (1974) were used. The range of hydraulic variables of the data are given in Table 1. Substituting the evaluated constants in (18a)–(18e) resulted in the following equations for the dispersion parameters:

Conservative Pollutants

$$m = 1.2 + 49 \left(\frac{R}{x}\right)^{0.121} \left(\frac{R^2}{A}\right)^{0.32} \left(\frac{V}{\sqrt{gR}}\right)^{0.16} \qquad (20a)$$

$$n = 2 \left[1 + 0.031 \left(\frac{x}{V} \sqrt{\frac{g}{R}}\right)^{0.27} \right]^{-1} \qquad (20b)$$

$$t_x = \frac{0.41x^{1.2}}{(gR)^{0.2}A^{0.1}V^{0.6}} \qquad (20c)$$

$$t_p = \frac{0.49x^{1.2}}{(gR)^{0.2}A^{0.1}V^{0.6}} \qquad (20d)$$

Nonconservative Pollutants

$$m = 1 + 16 \left(\frac{R}{x}\right)^{0.26} \left(\frac{\sqrt{gR}}{V}\right)^{0.2} \qquad (21a)$$

$$n = \left[1 + 0.4 \left(\frac{V}{\sqrt{gR}}\right)^{0.5} \right]^{-1} \qquad (21b)$$

$$t_x = \frac{0.5x^{1.2}}{(gR)^{0.2}A^{0.1}V^{0.6}} \qquad (21c)$$

$$t_p = \frac{0.56x^{1.2}}{(gR)^{0.2}A^{0.1}V^{0.6}} \qquad (21d)$$

$$t_d = \frac{3.4x^{0.9}A^{0.34}}{(gR)^{0.175}R^{0.58}V^{0.65}} \qquad (21e)$$

Figs. 3–7 depict the agreement of (20a)–(20d) and (21a)–(21e) with the experimental data of Nordin and Sabol (1974). It can be seen that the majority of data lies in the error band width of ±25%. This clearly reflects the utility of these equations. Eqs. (21a)–(21e) were obtained on the basis of data reported by Nordin and Sabol (1974). As such, these equations would be valid for streams having a nature similar to the general characteristics of the streams involved in field experimentation. Further, it needs to be pointed out that the decay time

TABLE 1. Range of Data

Parameters (1)	Conservative		Nonconservative	
	Minimum (2)	Maximum (3)	Minimum (4)	Maximum (5)
Average velocity, V (m/s)	0.04	1.51	0.05	1.06
Flow area, A (m²)	2.55	18,149.00	4.69	396.93
Hydraulic radius, R (m)	0.19	23.24	0.32	2.39
Distance from injection point, x (km)	2.57	294.45	13.44	135.32
Peak concentration, c (ppb)	0.34	124.12	1.05	40.96
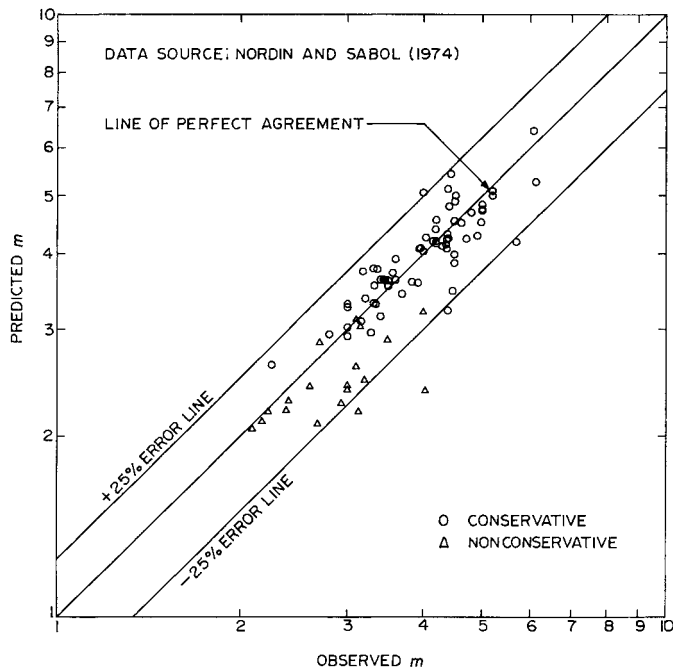
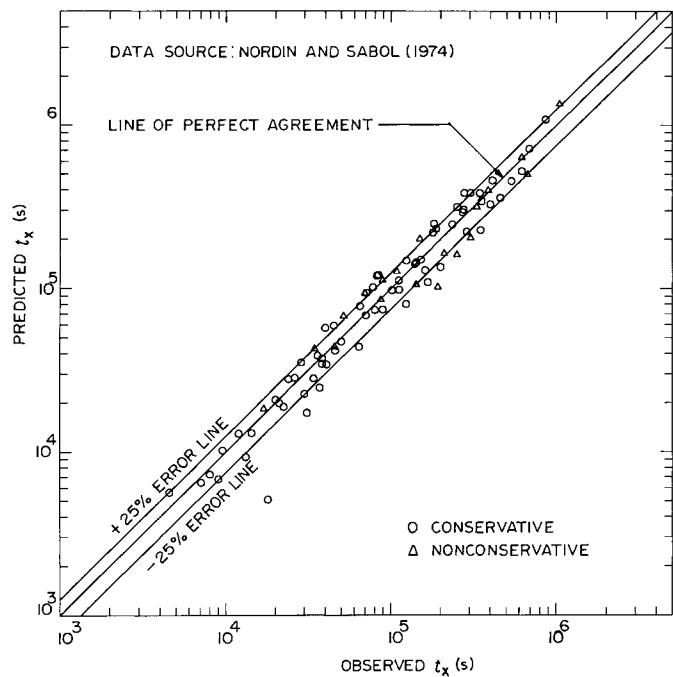**FIG. 3. Agreement of *m***

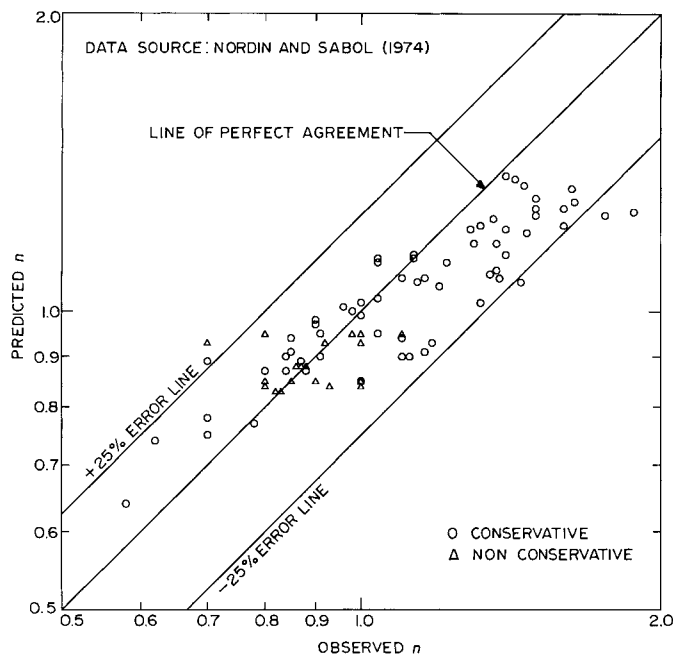**FIG. 5. Agreement of t_x**
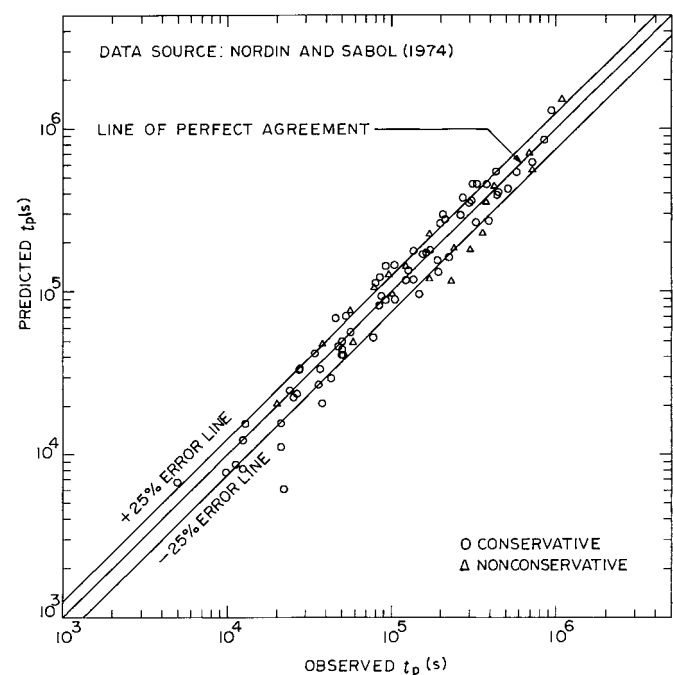

FIG. 4. Agreement of *n*

FIG. 6. Agreement of t_p

t_d as given by (21e) is strongly dependent on the reaction kinetics of the pollutant. Thus, (21e) is pollutant specific and is valid for the pollutant used in the experimental data reported by Nordin and Sabol (1974).

A perusal of (20c) and (20d) revealed that for conservative pollutants the peak concentration time lags by 20% of the inception time; i.e.

$$t_p = 1.2t_x \qquad (22a)$$

On the other hand, for the nonconservative pollutant, (21c) and (21d) indicate that the t_p lags by 12% of the t_x; i.e.

$$t_p = 1.12t_x \qquad (22b)$$

Using (9), (21c), and (21e), the total mass passing through a section was found as

$$M_x = M_o \exp\left[-\frac{x^{0.3}R^{0.58}}{6.8A^{0.44}}\left(\frac{V}{\sqrt{gR}}\right)^{0.05}\right] \qquad (23)$$

VALIDATION

The concentration equation (16) has been validated for conservative and nonconservative pollutants.

Conservative Pollutants

The field data of Taylor (1970) for Monocacy river at two sites has been used. The relevant geometric and hydraulic parameters are listed in Table 2. The dispersion parameters were computed from (20a)–(20d). Using these parameters, (16) was reduced to the following equations for sites 1 and 2, respectively:
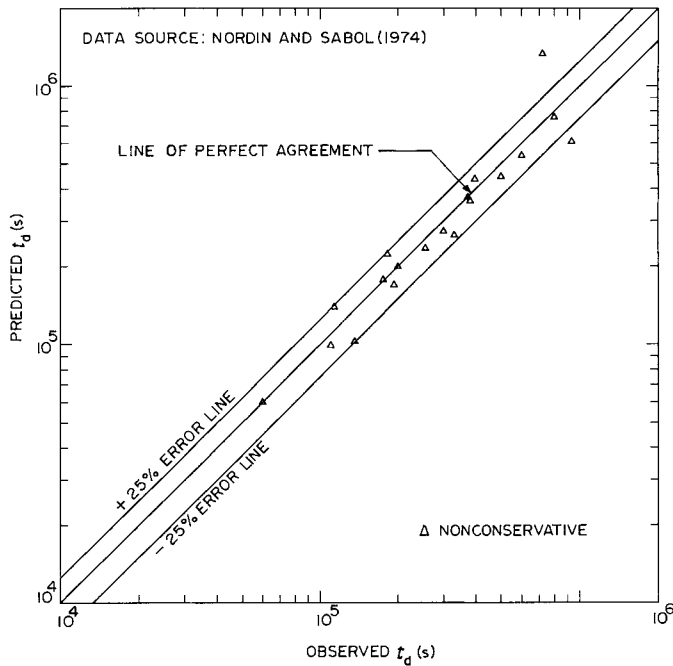
FIG. 7. Agreement of t_d

FIG. 8. Observed and Predicted c-t Curves (Conservative)

TABLE 2. Field Data and Dispersion Parameters

Entity (1)	Conservative		Nonconservative	
	Site 1 (2)	Site 2 (3)	Site 1 (4)	Site 2 (5)
Distance x (km)	18.343	34.272	27.755	33.789
Flow area A (m²)	28.79	33.03	37.58	35.06
Hydraulic radius R (m)	0.74	0.81	0.65	0.65
Average velocity V (m/s)	0.48	0.51	0.59	0.63
Discharge Q (m³/s)	15.57	15.57	22.07	22.07
Peak concentration c_p (ppb)	11.54	7.22	11.9	9.9
Exponent n	1.25	1.12	0.93	0.86
Inception time t_x (day)	0.473	0.926	0.826	1.011
Time to peak t_p (day)	0.567	1.097	0.917	1.125
Decay time t_d (day)	∞	∞	1.773	1.978

$$c = 12.8 \left(\frac{t - 0.46}{0.1}\right)^{3.25} \left[0.59 + 0.41 \left(\frac{t - 0.46}{0.1}\right)^{3.73}\right]^{-2.14} \quad (24a)$$

$$c = 6.9 \left(\frac{t - 0.92}{0.18}\right)^{3.11} \left[0.60 + 0.40 \left(\frac{t - 0.92}{0.18}\right)^{3.64}\right]^{-2.13} \quad (24b)$$

Fig. 8 depicts (24a) and (24b) along with the data points of Taylor (1970). Fig. 8 also shows c-t curves obtained by the Day and Wood (1976), Beltaos (1980), and Singh et al. (1992) methods. A perusal of Fig. 8 indicated that whereas these methods show large departures from the field data, the present method faithfully represents the data.

Nonconservative Pollutants

In this case also the field data of Taylor (1970) for Monocacy river were used in which the Rhodamine BA dye as a nonconservative pollutant (in which adsorption was the main mechanism for the loss of mass) was injected. The relevant geometric and hydraulic parameters are listed in Table 2. With further use of (21a)–(21e), the dispersion parameters were obtained. Using these parameters in (16), the following equations were obtained for sites 1 and 2, respectively:
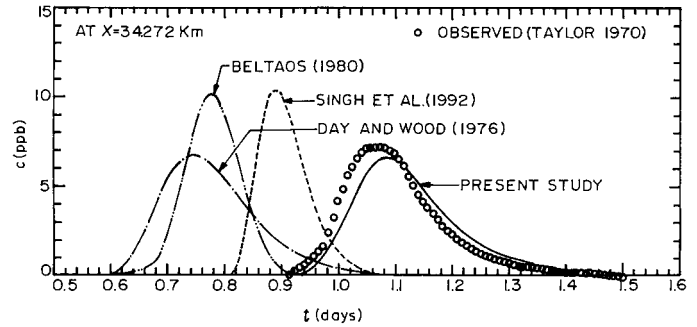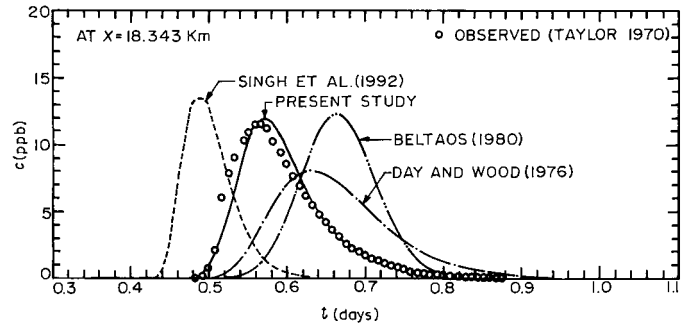
$$c = 11.63 \left(\frac{t - 0.82}{0.1}\right)^{1.34} \left[0.74 + 0.26 \left(\frac{t - 0.82}{0.1}\right)^{2.79}\right]^{-1.84} \quad (25a)$$

$$c = 8.25 \left(\frac{t - 1.05}{0.13}\right)^{1.26} \left[0.75 + 0.25 \left(\frac{t - 1.05}{0.13}\right)^{2.72}\right]^{-1.83} \quad (25b)$$

Fig. 9 shows (25) along with the data points of Taylor (1970). It can be seen that there is a fairly good agreement between the data and (25). The c-t curves obtained by the methods of Bansal (1971) and O'Loughlin and Bowmer (1975) have also been shown in Fig. 9. These curves show large deviations from the data points.

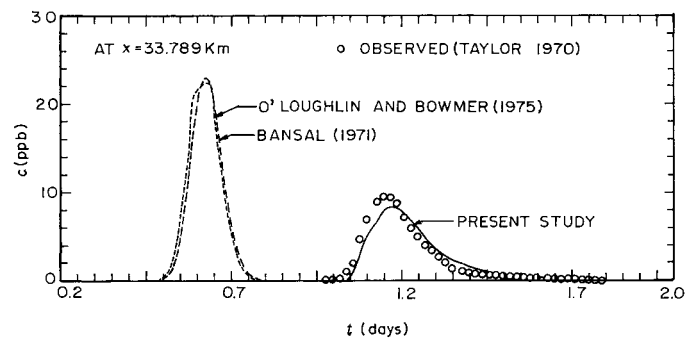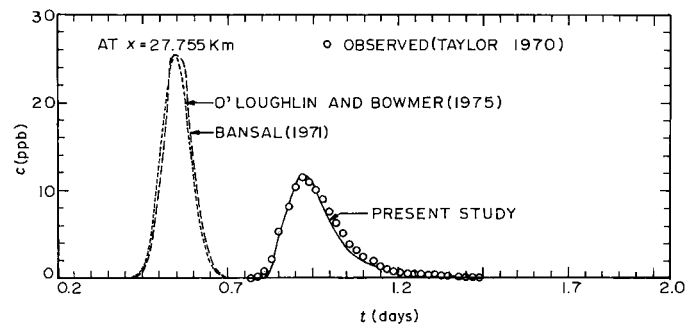Using (23), the total masses of pollutant at sites 1 and 2

FIG. 9. Observed and Predicted c-t Curves (Nonconservative)

were found to be 3.953 and 3.779 kg, respectively. The total masses at sites 1 and 2 as reported by Taylor (1970) are 3.919 and 3.87 kg, respectively. However, the total masses obtained by Bansal (1971) and O'Loughlin and Bowmer (1975) are 5.128 and 4.913 kg, respectively, and 5.059 and 4.901 kg, respectively. These values are far apart from the observed values.

VELOCITY OF PROPAGATION OF PEAK CONCENTRATION

Eqs. (20d) and (21d) were rearranged to give the location of peak concentration x_p in the following form:

$$x_p = \left[\frac{(gR)^{0.2}A^{0.1}V^{0.6}t_p}{\lambda} \right]^{0.833} \tag{26}$$

where $\lambda = 0.49$ for conservative pollutant and 0.56 for non-conservative pollutant of the present study. Differentiating x_p in (26) with respect to t_p gave the following equation for the velocity of propagation of the peak concentration:

$$V_p = \frac{\partial x_p}{\partial t_p} = \frac{5}{6} \left(\frac{gRV^3A^{0.5}}{\lambda^5 t_p} \right)^{1/6} \tag{27}$$

Eliminating t_p between (26) and (27) V_p was expressed in the following form involving x_p:

$$V_p = \frac{5}{6\lambda} \left(\frac{gRV^2A^{0.5}}{x_p} \right)^{0.2} \tag{28}$$

Eqs. (27) and (28) indicate that, at the time of mass injection, V_p is infinity and it progressively decreases with time and distance.

PRACTICAL APPLICATIONS

The practical application of the present study lies in solving both direct and inverse problems. The direct problem consists of predicting the hydraulic parameters and the c-t curve at distance x for known pollutant input M_o. This problem can be solved using (16) in conjunction with (20a)–(20d) or (21a)–(21e).

The inverse problem consists of obtaining the pollutant input M_o, its location, and the time of introduction. For this purpose concentration profiles at two locations are required. Denoting the locations by 1 and 2, the unknown distance of location 1 from pollution source is x_1. On the other hand, the distance between the pollution source and location 2 is $x_1 + \Delta x$, where Δx = known distance between the two locations. The unknown time of the peak concentration at location 1 is t_{p1} and that at location 2 is $t_{p1} + \Delta t_p$, where Δt_p = known time lag between the two peaks. Using (20d) or (21d) at the two locations, the lag time was expressed as

$$\Delta t_p = \frac{\lambda(x_1 + \Delta x)^{1.2}}{(gR_2)^{0.2}A_2^{0.1}V_2^{0.6}} - \frac{\lambda x_1^{1.2}}{(gR_1)^{0.2}A_1^{0.1}V_1^{0.6}} \tag{29}$$

Eq. (29) can be solved for x_1 by trial and error. Knowing x_1, t_{p1} can be obtained using (20d) or (21d). Thus, both the location and corresponding time of the pollutant input becomes known.

Rearranging (15a), M_o was expressed as

$$M_o = \frac{(n+1)(t_p - t_x)Qc_p}{m+n} \left[\frac{m(n+1)}{n(m-1)} \right]^n \exp(t_x/t_d) \tag{30}$$

where the dispersion parameters m and n are obtainable from the concentration profile (Fig. 2). On the other hand, knowing x_1 and $x_2 (=x_1 + \Delta x)$, as explained in the preceding paragraph, the corresponding parameters t_{x1} and t_{x2} can be obtained from (20c) or (21c). Similarly, t_{p1} and t_{p2} can be obtained from (20d)

or (21d). Further, t_{d1} and t_{d2} are obtainable from (21e). Using the dispersion parameters at stations 1 and 2, and (30), two estimates of the input mass M_o are obtained—M_{o1} and M_{o2}. The expected value of M_o is given by

$$E(M_o) = 0.5(M_{o1} + M_{o2}) \tag{31}$$

EXAMPLES

The solution procedure of the inverse problem is illustrated below through examples, for which the data of Taylor (1970) as given in Table 2 were used. Analyzing the concentration curves, the dispersion parameters at two sites were obtained. See Table 2.

Conservative Pollutants

Here $\lambda = 0.49$; $\Delta x = 15.929$ km; $\Delta t_p = 0.53$ day. Using the data in (29) yielded $x_1 = 18.65$ km by trial and error, which is very close to the actual distance 18.343 km (Taylor 1970). Thus, $x_2 = 18.65 + 15.929 = 34.579$ km.

Further, using the data corresponding to site 1 in (20d) yielded $t_{p1} = 0.565$ day. The observed $t_{p1} = 0.567$ day (Taylor 1970), which is very close to the predicted value.

With these data (30) yielded $M_{o1} = 1.829$ kg and $M_{o2} = 2.035$ kg. Eq. (31) yielded $E(M_o) = 1.932$ kg, which is close to the injected mass 1.904 kg (Taylor 1970).

Nonconservative Pollutants

In this case the observed data for site 1 are $\Delta x = 6.034$ km and $\Delta t_p = 0.208$ day. Adopting $\lambda = 0.56$ and solving (29) by trial and error gave $x_1 = 27.93$ km, which is very close to the actual distance 27.755 km (Taylor 1970). See Table 2.

Further, using (21d) $t_{p1} = 0.925$ day. The observed $t_{p2} = 0.917$ day (Taylor 1970), which is very close to the predicted value.

Using (30) and data gave $M_{o1} = 6.318$ kg and $M_{o2} = 6.729$ kg. Using these values (31) gave $E(M_o) = 6.524$ kg, which is also close to the injected mass 6.294 kg (Taylor 1970).

CONCLUSIONS

An empirical equation for predicting the c-t curve for an instantaneous injection of a mass of pollutant has been obtained. Predictor equations for the parameters involved in the equation have been related to hydraulic parameters and geometry. Using the equations it is possible to locate the source of pollution and its strength.

APPENDIX I.   REFERENCES

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APPENDIX II. NOTATION

The following symbols are used in this paper:

A = flow area;
c = mean concentration at section;
c_p = peak concentration;
D_L = longitudinal dispersion coefficient;
D_R = regional dispersion coefficient;
E = average percentage error;
g = gravitational acceleration;
K = decay constant;
K_d = decay constant;
k_0-k_{20} = constants;
L = characteristic length;
M = pollutant mass at time t;
M_o = injected mass;
M_p = cumulative mass at time t_p;
M_x = total mass of pollutant;
m = exponent;
N = number of observations;
n = exponent;
P = flow perimeter;
Q = discharge;
R = hydraulic radius;
T = flow width;
t = time;
t_* = time parameter;
t_d = decay time;
t_x = inception time;
t_p = time to peak;
V = average flow velocity;
V_p = velocity of propagation of peak concentration; and
x = longitudinal distance.