

# PREDICTING TRAVEL TIME AND DISPERSION IN RIVERS AND STREAMS

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**ABSTRACT:** The possibility of a contaminant being accidentally or intentionally spilled in a river is a constant concern to those using the water. Methods are developed to estimate: (1) the velocity of a contaminant in a river; (2) the rate of attenuation of the peak concentration of a conservative contaminant; and (3) the time required for a contaminant plume to pass a point. The methods are based on data collected by the U.S. Geological Survey in almost a hundred different rivers representing a wide range of sizes, slopes, and geomorphic types. Although the accuracy of the predictions can be greatly increased by performing time-of-travel studies, the emphasis of this paper is on providing methods for making estimates where few data are available. It is shown that the unit-peak concentration is well correlated with travel time and that the travel time of the leading edge averages 89% of the travel time of the peak concentration.

## INTRODUCTION

The possibility of a contaminant being accidentally or intentionally spilled upstream from a water intake is a constant concern to those diverting and using water from streams and rivers. A method of rapidly estimating travel time and dispersion is needed for pollution control or warning systems on streams where data are limited. Travel time and mixing of water within a stream are also basic streamflow characteristics that hydrologists should understand in order to predict the rate of movement and dilution of pollutants that may be introduced into streams.

With the widespread availability of computers today, it is natural to think of numerical models as a means of addressing these questions. Although many excellent models are available to make the calculations needed to estimate travel time and dispersion, none can be used with confidence before calibration and verification to the particular river reach in question. That is, all models must be provided with information from which flow velocities and mixing rates can be computed. In general, there are no reliable methods of predicting dispersion coefficients (mixing rates) from commonly available hydraulic information and the very detailed channel geometry data and flow resistance coefficients needed to predict stream velocities are seldom available. The availability of reliable flow velocities and mixing rates is, therefore, almost always the weakest link in the chain of events needed to predict the rate of movement, dilution, and mixing of pollutants in rivers and streams.

Soluble tracers can be used to simulate the transport and dispersion of solutes in surface waters because they have virtually the same physical characteristics as water (Feurstein and Selleck 1963; Smart and Laidlaw 1977). This is the case in either a steady flowing river or in the unsteady oscillatory stage and flow of a tidal estuary. Measured tracer-response curves produced from the injection of a known quantity of soluble tracer provides an efficient method of obtaining the data necessary to calibrate and verify pollutant transport models. These data can also be used, in conjunction with the superposition principle, to simulate pollution buildup in streams, lakes, and estuaries without the need to use numerical models.

Extensive use of fluorescent dye tracers to quantify the transport and dispersion in streams and rivers began in the United States in the early to mid-1960s. Kilpatrick (1993), developed the concept of unit-peak concentration and used the superposition principle to illustrate how data obtained in time-

of-travel studies could be generalized to a wide range of flow conditions and even to other sites.

In this paper, the concepts presented by Kilpatrick (1993), along with extensive data collected by the U.S. Geological Survey on time of travel and dispersion, are used to develop methods to estimate: (1) the rate of movement of a solute through a river reach; (2) the rate of attenuation of the peak concentration of a conservative solute with time; and (3) the length of time required for the solute plume to pass a point in the river. This information is required to calibrate or verify pollutant transport models. The accuracy of these predictions will be increased greatly by performing time-of-travel studies on the river reach in question; but the emphasis of this paper is on providing methods for making estimates in rivers where few data are available. Travel time and concentration attenuation of pollutants not dissolved in the water are beyond the scope of this paper.

This paper begins with a short discussion of the theory of movement and dispersion of dissolved pollutants and introduces the unit-peak concentration concept. Methods are recommended for estimating the rate of movement and attenuation of conservative pollutants based on an analysis of the data compiled by Jobson (1996). The paper concludes by illustrating the application of these methods by use of two examples.

## BACKGROUND AND TECHNIQUES

### Theory of Transport and Dispersion for Instantaneous Sources

The response to the slug injection of a soluble tracer is assumed to imitate the characteristics of a soluble pollutant, so understanding of how tracers mix and disperse in a stream is essential to understanding their application in simulating pollution. Time-of-travel studies are often conducted to help understand these processes and to quantify travel time and dispersion for a given reach of river. The general procedure for conducting a time-of-travel study is to instantaneously inject a known quantity of water-soluble tracer into a stream, usually at the center of flow, and to observe the variation in concentration of the tracer as it moves downstream. The general distribution of a tracer concentration resulting from a slug injection is shown in Fig. 1. The tracer-response curves in Fig. 1 are shown as a function of longitudinal distance and not as a function of time. Later in the paper, the response curves will generally be shown as a function of time.

Dispersion and mixing of a tracer in a receiving stream take place in all three directions (Fig. 1). In this paper, vertical and lateral diffusion will be referred to in a general way as mixing. The elongation of the tracer-response cloud longitudinally will be referred to as longitudinal dispersion. Vertical mixing is completed rather rapidly, normally within a distance of less than 50 river depths (1–10 widths) downstream of the injection point.

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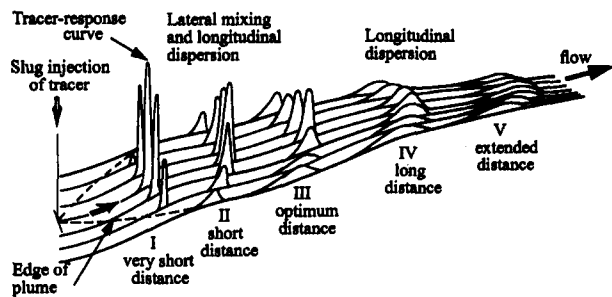


FIG. 1. Lateral Mixing, Longitudinal Dispersion Patterns, and Changes in Distribution of Concentration Downstream from Single, Center, Slug Injection of Tracer [Modified from Kilpatrick (1993), p. 2]

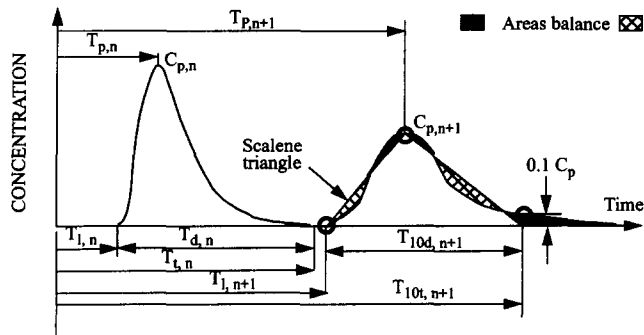


FIG. 2. Definition Sketch for Tracer-Response Curves—Symbols Are Explained in Text [Modified from Kilpatrick and Wilson (1989), p. 3]

tion (Rutherford 1994, p. 70). Lateral mixing is much slower but is usually complete within 100–300 river widths (Rutherford 1994, p. 123). Longitudinal dispersion, having no boundaries, continues indefinitely. In other words, vertical mixing is likely to be complete at section I in Fig. 1, which is a very short distance downstream of the injection. At section II lateral mixing is still taking place rapidly, so mixing and dispersion are both significant processes between the injection and section III on Fig. 1. Downstream of section III the dominant mixing process is longitudinal dispersion, so the tracer concentration can generally be assumed to be uniform in the cross section.

For a midpoint injection, the tracer cloud moves faster than the mean stream velocity upstream of section III because the bulk of the tracer is in the high velocity part of the cross section. Preferably, all measurement cross sections for a time-of-travel study are at least as far downstream as the optimum distance (section III in Fig. 1) so that longitudinal dispersion is the dominant process acting between measurement cross sections and so the tracer moves downstream at the mean stream velocity.

The conventional manner of displaying the response of a stream to a slug injection of tracer is to plot the variation of concentration with time (the tracer-response curve) at two or more cross sections downstream of the injection, as illustrated on Fig. 2. The tracer-response curve, defined by the analysis of water samples taken at selected time intervals during the tracer-cloud passage is the basis for determining time-of-travel and dispersion characteristics of streams. A detailed explanation of the analysis and presentation of time-of-travel data are covered in the report by Kilpatrick and Wilson (1989). A much less detailed explanation is given by Rutherford (1994).

The characteristics of the tracer-response curves shown in Fig. 2 are described in terms of elapsed time after an instantaneous tracer injection, where  $C_p$  = peak concentration of the tracer cloud;  $T_l$  = elapsed time to the arrival of the leading edge of a tracer cloud at a sampling location;  $T_p$  = elapsed

time to the peak concentration of the tracer cloud;  $T_t$  = elapsed time to the trailing edge of the tracer cloud;  $T_d$  = duration of the tracer cloud ( $T_t - T_l$ );  $T_{10d}$  = duration from leading edge until tracer concentration has reduced to within 10% of the peak concentration; and  $n$  = number of sampling site downstream of injection.

The mass of tracer to pass a cross section,  $M_r$ , is computed as

$$M_r = \int_{T_l}^{T_t} \int_0^W C_v \times q \times dw \times dt \quad (1)$$

where  $W$  = total width of the river;  $C_v$  = vertically averaged tracer concentration; and  $q$  = unit discharge (discharge per unit width). Both  $C_v$  and  $q$  are given at time  $t$  and distance  $w$  from one bank. After mixing is complete in the cross section, the equation simplifies to

$$M_r = \int_{T_l}^{T_t} C \times Q \times dt \quad (2)$$

where  $C$  is assumed to be uniform in the cross section and  $Q$  is the total discharge in the cross section at time  $t$ . If mixing is not complete, (2) can still be used as long as the concentration  $C$  is the discharge-weighted, cross-sectional-average concentration. If discharge is constant during the passage of a tracer cloud, it can also be factored out of the integral.

The shape and magnitude of the observed tracer-response curves shown in Figs. 1 and 2 are determined by four factors: (1) the quantity of tracer injected; (2) the degree to which the tracer is conservative; (3) the magnitude of the stream discharge; and (4) longitudinal dispersion. All of these factors must be taken into consideration to predict the concentration of solutes from tracer-concentration data.

The magnitude of the tracer concentration in a stream is in direct proportion to the mass of tracer injected,  $M_i$ . Doubling the amount of injected tracer for a given flow will double the observed concentrations, but the shape and duration of the tracer-response curve will remain constant. For this reason most investigators normalize their data by dividing all observed tracer concentrations by the mass of tracer injected,  $M_i$  (Bailey et al. 1966; Martens et al. 1974).

It has also been found that various tracers are lost in transit due to adhesion on sediments and photochemical decay. Scott et al. (1969) found fluorescent dyes to be absorbed on fine sediments such as clay. Rhodamine WT dye has been shown both in the field and laboratory to decay photochemically about 2–4% per day (Hetling and O'Connell 1966; Tai and Rathbun 1988). Kilpatrick (1993) noted decay rates tended to be higher in rivers, about 5% per day, compared to about 3% per day in estuaries.

To compare data and to have it simulate a conservative substance, it is desirable to eliminate the effects of tracer loss. If the stream discharge,  $Q$ , is measured at the same time and location as the tracer concentration, it is possible to evaluate the mass of tracer recovered,  $M_r$ , from (1) or (2). When the mass of the tracer injected,  $M_i$ , is known, the tracer recovery ratio  $R_r$  can be expressed as

$$R_r = \frac{M_r}{M_i} \quad (3)$$

A factor that inversely affects the magnitude of the tracer-response curves is the stream discharge. The diluting effect of tributary inflows, as well as that of natural ground-water accretion, differs from stream to stream and with location on the same stream. To counter the variable diluting effects of differing discharges, it is desirable to adjust observed concentration data by multiplying by the stream discharge.

Observed concentrations can be adjusted for: (1) the amount of tracer injected; (2) tracer loss; and (3) stream discharge (three of the four factors affecting the concentration) by use of what is called a unit concentration. The unit concentration is defined as 1,000,000 times the observed concentration times the river discharge divided by the total mass to pass the cross section. It is computed by the equation

$$C_u = 1 \times 10^6 \times \frac{C}{R_r} \times \frac{Q}{M_i} = 1 \times 10^6 \times \frac{C}{M_r} \times Q \quad (4)$$

Unit concentration ( $C_u$  with units of inverse time) can be visualized as the conservative mass transport rate of solute (milligrams per liter times liters per second = milligrams per second) per unit of mass injected (milligrams). The 1,000,000 simply makes the numbers closer to unity. The discharge must be expressed in units that are consistent with the denominator of the concentration, and the injected mass must be in the same units as the numerator of the concentration. For example, if the concentration is expressed in milligrams per liter, the injected mass must be expressed in milligrams and the discharge must be expressed in liters per unit time. If the entire tracer cloud is sampled, the value of  $M_r$  can be computed and the mass of injected tracer need not be known.

Eq. (4) can be used to convert any measured tracer-response curve to a unit-response curve. This unit-response curve is a direct measure of the dispersion and can be used as the building block for simulating the concentrations to be expected from various pollutant loadings at different stream discharges. When the flow is constant and mixing is complete, the area under unit-response curves is constant ( $1 \times 10^6$ ) for any cross section on a stream.

## ANALYSIS OF EXISTING DATA AND DEVELOPMENT OF PREDICTION EQUATIONS

### Attenuation of Unit-Peak Concentration

Mixing processes have usually been interpreted by use of the Fickian theory of diffusion, and Fischer (1967) used this theory to define longitudinal dispersion coefficients for mixing in rivers. The peak concentration is a very important point on a tracer-response curve, and its variation with lapsed time since injection is a direct measure of the efficiency of the mixing process. According to Fischer's dispersion model, the peak concentration should attenuate with time as

$$C_{up} \propto t^{-\beta} \quad (5)$$

in which  $C_{up}$  = unit-peak concentration;  $t$  = time since injection; and  $\beta$  = coefficient. The value of  $\beta$  should be approximately 1.5 for very short dispersion times (section I in Fig. 1) and decrease to 0.5 for very long dispersion times (section V in Fig. 1). Nordin and Sabol (1974) argue that a Fickian type equation cannot adequately describe longitudinal dispersion in rivers because the value of  $\beta$  never decreases to a value of 0.5. They conclude that a typical value of  $\beta$  is 0.7.

After mixing in the cross section is complete, the decrease of the unit-peak concentration with time (as measured by  $\beta$ ) is a measure of the longitudinal mixing efficiency. Larger values of  $\beta$  indicate more rapid longitudinal mixing. The presence of pools and riffles, bends, and other channel and reach characteristics will increase the rate of longitudinal mixing (perhaps by contributing to dead zones) and almost always yield a value of  $\beta$  greater than the Fickian value of 0.5.

Unit-peak concentrations have been compiled for 422 cross sections obtained from more than 60 different rivers in the United States (Jobson 1996). These data represent mixing conditions in rivers with a wide range of size, slope, and geomorphic type. For example, the slope in the study reach of the

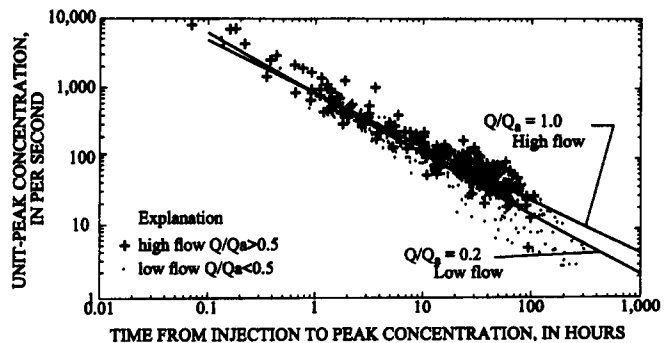


FIG. 3. Unit Concentrations as Function of Traveltime with Eq. 7 Plotted on Figure for Two Values of  $Q/Q_a$

Mississippi River, near Baton Rouge, La. is 0.01 m/km and the mean annual discharge is about 11,000 m<sup>3</sup>/s, whereas the study reach of Bear Creek, near Morrison, Colo., has a slope of 36.0 m/km and a mean annual discharge of only about 1.3 m<sup>3</sup>/s.

Fig. 3 is a plot of the unit-peak concentrations ( $C_{up}$ ) as a function of elapsed time ( $T_p$ ) to the peak of all the data. A tight correlation is shown by the data, indicating that a reasonable estimate of the unit-peak concentration can be determined from an expression of the form of (5). The regression equation based only on traveltime that best fit all of the data was

$$C_{up} = 1,025 \times t_p^{-0.887} \quad (6)$$

This equation predicted the 422 available data points with a root mean square (RMS) error of 0.502 natural log units. The coefficient of variation was 0.112 and the coefficient of determination ( $R^2$ ) value was 0.893. The standard error of estimate of the coefficient is 4.9% and the standard error of estimate for the exponent is 1.7%.

Other river characteristics that were available to help define the relation included the drainage area ( $D_a$ ), the reach slope ( $S$ ), the mean annual river discharge ( $Q_a$ ), and the discharge at the time of the measurement ( $Q$ ). The most significant other variable in the correlation was the relative discharge ( $Q/Q_a$ ) giving a prediction equation

$$C_{up} = 857 T_p^{-0.760(Q/Q_a)^{-0.079}} \quad (7)$$

Data were available with relative discharges ranging from 0.01 to 7.8. The median, 75 and 25 percentile values were 0.41, 0.82, and 0.20, respectively. Eq. (7) predicted the 410 data points, which had mean annual flow available, with an RMS error of 0.426 natural log units. The coefficient of variation was 0.100 and the  $R^2$  value was 0.910. The standard error of estimate of the coefficient is 4.3%, and the standard error of estimate for the exponent (0.760) is 1.6%.

The data in Fig. 3 are separated into two groups—one with values of relative discharge greater than 0.5 (high flow) and one with a relative discharge less than 0.5 (low flow). The lines for high flow and low flow are plotted assuming constant values of relative discharge of 1.0 and 0.2, the approximate median value for each group of data.

Slope was not significant as an explanatory variable. Various regression models based on different combinations of discharge, mean annual discharge, and drainage area were tried. None of the equations produced a smaller RMS error or a larger  $R^2$  value than (7).

Results for individual rivers generally define a much closer relation. For example, Fig. 4 presents measured concentrations of dye for the Shenandoah River as published by Taylor et al. (1986). The points labeled as  $Q/Q_a = 0.65$  were actually taken at relative discharges ranging from 0.57 to 0.79 and the points labeled as  $Q/Q_a = 0.27$  actually ranged from 0.21 to 0.32.

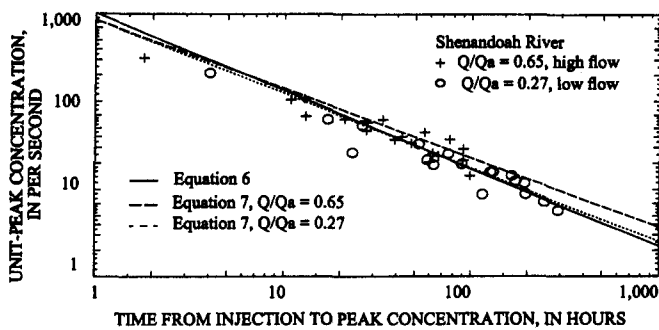


FIG. 4. Unit-Peak Concentrations of Dye for Shenandoah River

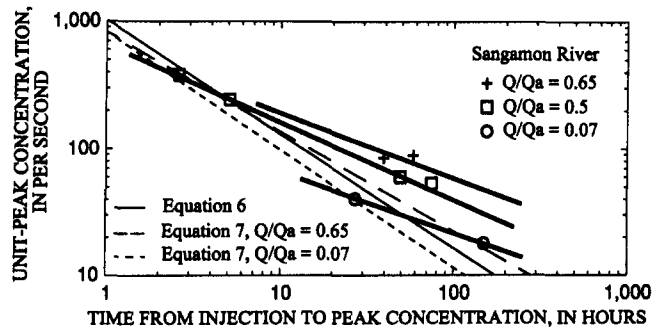


FIG. 5. Unit-Peak Concentrations of Dye for Sangamon River

Notice that the data for the Shenandoah River show almost no correlation with relative discharge. Eqs. (6) and (7) are also plotted on the figure for reference. In this case the equations fit the data very closely.

The Sangamon River data (Graf 1986) shows strong correlations with relative discharge (Fig. 5). It should be noted, however, that one set of measurements was made at extremely low flow. The scatter among points for a single river is typically much less than the scatter among all rivers (Fig. 3) so there is significant value in collecting data for individual rivers to improve the ability to predict the variation of unit-peak concentration.

The more efficient the mixing in a river, the steeper will be the relation between unit-peak concentration and traveltime. At high flow, river channels generally tend to be relatively uniform in shape, and they tend to increasingly exhibit a pool and riffle structure as the flow decreases. A pool and riffle structure offers great opportunities for tracer trapping; therefore, a pool and riffle structure tends to be efficient in mixing and attenuating the peak concentration. Eq. (7) accounts for this process by changing the slope of the curve with relative discharge.

### Time-of-Travel of Peak Concentration

As shown in the preceding section, the time required for a tracer cloud to reach a specific point in a river is the dominant factor in determining the concentration. Traveltime itself is also of interest to local planners, who may be more interested in the minimum probable traveltime than the expected traveltime. Water velocity depends on many factors including the general morphology of the river and particularly the amount of ponding caused by dams or other man-made works. Prediction of the traveltime is, therefore, very important and it is often more difficult than the prediction of unit-peak concentration at a given traveltime.

Stream velocity and, consequently, traveltime commonly vary with discharge. The relation of mean stream velocity,  $V$ , to discharge is generally assumed to take the form

$$V = K \times Q^a \quad (8)$$

which is a straight line when the logarithm of discharge,  $Q$ , is plotted against the logarithm of velocity. For accurate estimates, both the constant,  $K$ , and exponent,  $a$ , must be defined for each river reach, so two or more time-of-travel measurements are required to define the transport characteristics of the river reach. Geomorphic analyses by many investigators suggest that the exponent in (8) ranges from about 0.2 to 0.5, but typically has a value of about 0.34 (Jobson 1989).

The velocity of the peak concentration and associated hydraulic data are compiled in Jobson (1996) for about 90 different rivers in the United States representing a wide range of river sizes, slopes, and geomorphic types. Four variables were available in sufficient quantities for regression analysis. These included the drainage area  $D_a$ , the reach slope  $S$ , the mean annual discharge  $Q_a$ , and the discharge at the section at time of the measurement,  $Q$ . It was reasoned that these variables could be combined into the following dimensionless groups. The dimensionless peak velocity is defined as

$$V'_p = \frac{V_p D_a}{Q} \quad (9)$$

The dimensionless drainage area is defined as

$$D'_a = \frac{D_a^{1.25} \times \sqrt{g}}{Q_a} \quad (10)$$

in which  $g$  = acceleration of gravity. The dimensionless relative discharge is defined as

$$Q'_a = \frac{Q}{Q_a} \quad (11)$$

These equations are homogeneous, so any consistent system of units can be used in the dimensionless groups. The regression equations that follow, however, have a constant term that has specific units, meters per second. A convenient set of units for use with the equations is, therefore, velocity in meters per second, discharge in cubic meters per second, drainage area in square meters, acceleration of gravity in meters per second squared, and slope in meters per meter.

The most accurate prediction equation, based on 939 data points where all variables were available, for the peak velocity in meters per second was

$$V_p = 0.094 + 0.0143 \times (D'_a)^{-0.919} \times (Q'_a)^{-0.469} \times S^{0.159} \times \frac{Q}{D_a} \quad (12)$$

The standard error of estimates of the constant and slope are 0.26 m/s and 0.0003, respectively. This prediction equation has an  $R^2$  of 0.70 and an RMS error of 0.157 m/s. Fig. 6 contains a plot of the observed velocities as a function of the variables on the right side of (12).

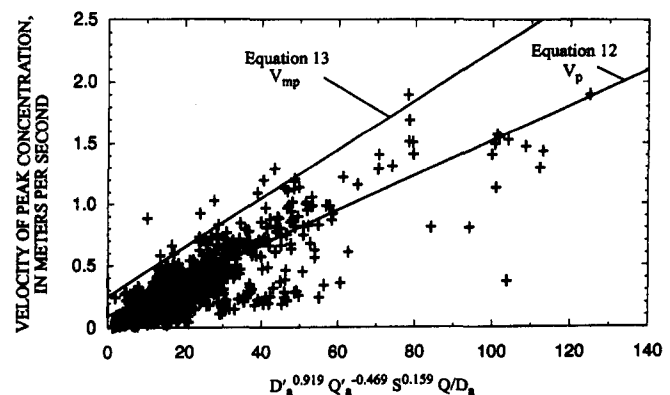


FIG. 6. Plot of Velocity of Peak Concentration as Function of Dimensionless Drainage Area, Relative Discharge, Slope, Local Discharge, and Drainage Area

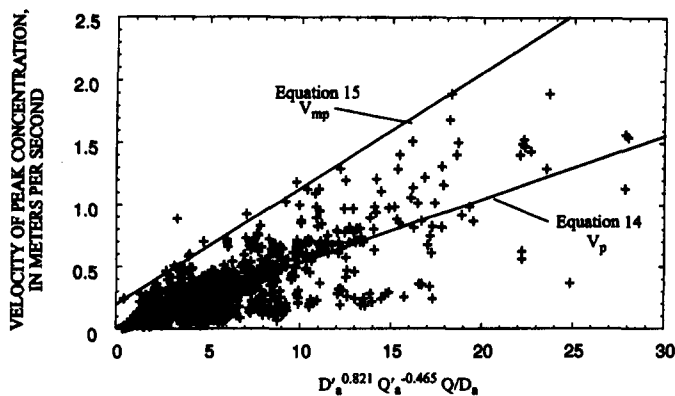


FIG. 7. Plot of Velocity of Peak Concentration as Function of Dimensionless Drainage Area, Relative Discharge, Local Discharge, and Drainage Area

For responses to accidental spills, the highest probable velocity, which will result in the highest concentration, is usually a concern. On Fig. 6 an envelope line for which more than 99% of the observed velocities are smaller is also shown. The equation for this line, the maximum probable velocity, in meters per second,  $V_{mp}$ , is

$$V_{mp} = 0.25 + 0.02 \times (D_a')^{0.919} \times (Q_a')^{-0.469} \times S^{0.159} \times \frac{Q}{D_a} \quad (13)$$

The best equation for the velocity of the peak, in meters per second, that did not include slope, which is often a difficult variable to estimate, was

$$V_p = 0.020 + 0.051 \times (D_a')^{0.821} \times (Q_a')^{-0.465} \times \frac{Q}{D_a} \quad (14)$$

The standard error of estimates of the constant and slope are 0.009 m/s and 0.0013, respectively. The root-mean-square error of the prediction equation, based on 986 points, is 0.17 m/s with an  $R^2$  of 0.62. Fig. 7 presents a plot of the observed velocities as a function of the variables on the right side of (14). Also shown on the figure is a line for which 99% of the data points indicate a smaller velocity. The equation for this line, for the probable maximum velocity, in meters per second, is

$$V_{mp} = 0.2 + 0.093 \times (D_a')^{0.821} \times (Q_a')^{-0.465} \times \frac{Q}{D_a} \quad (15)$$

### Time-of-Travel of Leading Edge

In addition to knowing when the peak concentration will arrive at a site, it is of great interest to know when the first

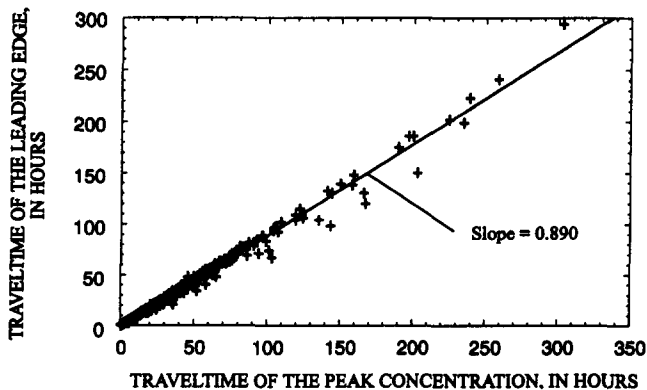


FIG. 8. Plot of Time From Injection to First Arrival of Leading Edge of Tracer Cloud as Function of Traveltime of Peak Concentration

pollutant will arrive. The time of arrival of the leading edge of the pollutant indicates when a local problem will begin and defines the overall shape of the concentration response function.

Fewer data were available for the time of arrival of the leading edge (520 sites) than were available for the velocity of the peak concentration. Eight variables were available in sufficient quantities for regression analysis. These included the drainage area  $D_a$ , the reach slope  $S$ , the mean annual discharge  $Q_a$ , the discharge at the section at time of the measurement  $Q$ , the velocity of the peak concentration  $V_p$ , the width of the river, the depth of the river, and the time from the injection to the passage of the peak concentration (traveltime of the peak concentration,  $T_p$ ). No significant correlation could be found between any of the variables and the time from injection to the arrival of the leading edge ( $T_l$ ) except for the traveltime to the peak concentration. Fig. 8 contains a plot of the traveltime of the leading edge as a function of the traveltime of the peak concentration. As can be seen from the figure, the correlation between these two variables is very good with an  $R^2$  of 0.989, a coefficient of variation of 0.13, and a RMS error of 3.78 hours. These data indicate that the traveltime of the leading edge can be estimated from

$$T_l = 0.890 \times T_p \quad (16)$$

### Time-of-Passage of Pollutant

Methods have been developed for estimating the traveltime of the leading edge,  $T_l$ , the traveltime of the peak concentration,  $T_p$ , and the magnitude of the unit-peak concentration,  $C_{up}$ . This information defines two points on the tracer-response curve, shown as two of the large dots on Fig. 2. When time is scaled by the width of the cloud, both Day and Wood (1976) and Rutherford et al. (1980) show that concentration response curves are strongly self similar. Using the self-similar concept, Kilpatrick and Taylor (1986) show that the area of a normal slug-produced tracer-response curve is very nearly equal to the area of a scalene triangle (three unequal sides) with a height equal to the peak concentration and the base extending from the leading edge to a point where the trailing edge concentration is equal to 0.1 times the peak concentration,  $T_{d10}$  (Fig. 2). Because the area under the unit-response curve is  $1 \times 10^6$ , this information can be used to estimate a third point on the curve. The time of passage from the leading edge to a point where the concentration has been reduced to 10% of the peak concentration,  $T_{d10}$ , can be estimated from the equation

$$T_{d10} = \frac{2 \times 10^6}{C_{up}} \quad (17)$$

When the time base of the tracer response curves are scaled according to (17), the normalized response curves given by both Day and Wood (1976) and Rutherford et al. (1980) contain a unit area to within a fraction of 1%. Even when the rising and falling limbs are scaled independently to pass through the three arbitrary points, the area under the curve deviates from unity by less than 2%. This allows a complete tracer-response curve to be sketched in with good accuracy using only the peak concentration and the times to the leading edge and peak.

### EXAMPLE APPLICATIONS

Two example applications for a slug injection will be given. The first example will assume that very few hydrologic data are available, and the second example will assume that time-of-travel measurements have been made at a relatively high and relatively low discharge.

### Example 1, Very Limited Data

Assume that a truck runs off the road and instantaneously spills 6,000 kg of a corrosive chemical into an ungauged stream. Estimate the most probable and the expected worst case effects of the spill on the water intake for a town that is located 15 km downstream. The worst case should occur for the shortest probable traveltime.

No data exist for the stream receiving the spill, but topographic maps show that the drainage area is 350 km<sup>2</sup> at the spill site and 430 km<sup>2</sup> at the intake for the town. A review of available data also indicates that a gauging station exists for a nearby stream with a drainage area of 452 km<sup>2</sup> and a mean-annual flow of 5.22 m<sup>3</sup>/s. At the time of the spill the flow at the gauging station was 3.88 m<sup>3</sup>/s. The hydrology and weather are assumed to be uniform so, assuming an average drainage area for the reach of (350 + 430)/2 = 390 km<sup>2</sup>, the discharge of the stream carrying the spill is estimated to be 3.88 (390/452) = 3.35 m<sup>3</sup>/s. Likewise, the mean-annual flow of the ungauged stream is estimated to be about 5.22 (390/452) = 4.50 m<sup>3</sup>/s.

The first step is to estimate the traveltime of the peak concentration. Because the river slope is not available, (14) and (15) will be used to estimate the expected and fastest probable traveltimes in the stream. The dimensionless drainage area and discharge are computed first from (10) and (11)

$$D'_a = \frac{(390 \times 10^6)^{1.25} \times \sqrt{9.8}}{4.50} = 3.81 \times 10^{10}$$

$$Q'_a = \frac{3.35}{4.50} = 0.744$$

Applying (14)

$$V_p = 0.020 + 0.051(3.81 \times 10^{10})^{0.821}(0.744)^{-0.465} \cdot [3.35/(390 \times 10^6)] = 0.264 \text{ m/s}$$

while the maximum probable velocity from (15) is

$$V_{mp} = 0.2 + 0.093(3.81 \times 10^{10})^{0.821}(0.744)^{-0.465} \cdot (3.35/390 \times 10^6) = 0.646 \text{ m/s}$$

The most probable traveltime of the peak to the water intake is

$$T_p = 15,000/(0.264 \times 3,600) = 15.8 \text{ hours}$$

and the probable minimum traveltime of the peak is

$$T_{pm} = 15,000/(0.646 \times 3,600) = 6.4 \text{ hours}$$

With the traveltimes known, the most probable unit-peak concentration at the town intake can be estimated from (7) as

$$C_{up} = 857 \times 15.8^{-0.760 \times 0.744^{-0.079}} = 100 \text{ per second}$$

Rearranging (4) to give the peak concentration

$$C_p = \frac{C_{up} \cdot R_r \cdot M_i}{1 \times 10^6 \cdot Q}$$

and using the injected mass,  $M_i$ , of  $6 \times 10^9$  mg, the flow rate at the intake,  $Q$ , of  $[3.88 \times (430/452) \times 1,000] = 3,690$  L/s, and assuming the recovery ratio,  $R_r$ , to be 1.0, the most probable conservative-peak concentration can be computed as

$$C_p = 100 \times 1.0 \times 6 \times 10^9 / (3,690 \times 10^6) = 162 \text{ mg/L}$$

occurring 15.8 hours after the injection.

At the highest probable velocity, the unit-peak concentration is  $202 \text{ s}^{-1}$  giving an estimated conservative peak concentration of 328 mg/L occurring 6.4 hours after the spill.

When will the pollutant first arrive at the intake? As can be seen from (16), the time of arrival of the leading edge of the pollutant cloud is predicted to occur  $0.89 \times 15.8 = 14$  hours after the accident. It is highly unlikely that the pollutant will arrive at the intake sooner than  $0.89 \times 6.4 = 5.7$  hours after the spill.

How long will the intake be affected? As can be seen from (17), the most probable time required for the bulk of the pollutant to pass the site (the concentration to be reduced to 10% of the peak value, 16 mg/L) is

$$T_{d10} = 2 \times 10^6 / (100 \times 3,600) = 5.6$$

hours after the time of arrival, or  $14 + 5.6 = 19.6$  hours after the spill.

It is highly unlikely that the pollutant concentration will have reduced to less than 20 mg/L before

$$5.7 + 2 \times 10^6 / (202 \times 3,600) = 8.5 \text{ hours after the spill}$$

All of these computations were carried out assuming no loss of pollutant between the spill and the intake. Losses could occur by chemical reactions, volatilization, absorption on the streambed, or other processes.

### Example 2, Traveltime Data Available

The second example assumes that 50 kg of a pollutant is spilled in the Apple River 25.9 km upstream of Elizabeth, Ill. (10 km from the injection site), when the discharge at the spill site is 2.4 m<sup>3</sup>/s. Compute the probable concentration, assuming no losses, of this spill at a water intake for Hanover, which is 41.1 km downstream of the spill.

Two time-of-travel studies have been completed on this reach of the Apple River and the data are presented in Graf (1986). One of these studies was conducted at relatively low flow, when the river discharge was about 0.7 times the mean annual flow, and one was conducted at relatively high flow, when the flow rate was about 3.5 times the mean annual flow. The mean annual discharge of the Apple River near Apple River, at Elizabeth, and at Hanover are 0.7, 3.8, and 5.0 m<sup>3</sup>/s, respectively (Jobson 1996, p. 46). The first step is to estimate the time of travel of the peak and leading edge of the pollutant cloud. The measured traveltimes of the peak concentrations are plotted in Fig. 9.

From Fig. 9 it is seen that the traveltime of the peak concentration to Elizabeth is 49.4 hours at a low relative discharge of 0.68, while the traveltime to Whitton is 105.8 hours at a relative discharge of 0.62. It is also seen that the distance from Elizabeth to Hanover is 16.1 km while the distance from Elizabeth to Whitton is 22.5 km, so Hanover is 71.6% of the way between Elizabeth and Whitton. By linear interpolation, it is easily seen that the traveltime from the injection site to

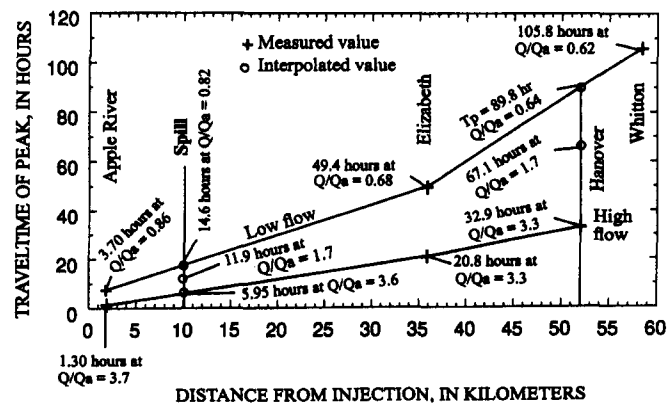


FIG. 9. Traveltime Distance Relation for Peak Concentration in Apple River

Hanover would be about  $49.4 + (105.8 - 49.4) \times 0.716 = 89.8$  hours and that the relative discharge at this point would have been about  $0.68 + (0.62 - 0.68) \times 0.716 = 0.64$ . Likewise, the spill site is 23.8% of the way between the town of Apple River and Elizabeth so the traveltime from Apple River to the spill site would be  $3.7 + (49.4 - 3.7) \times 0.238 = 14.6$  hours at a relative discharge of  $0.86 + (0.68 - 0.86) \times 0.238 = 0.82$ .

At high flow, the traveltime from the town of Apple River to the spill site could be estimated as 5.95 hours at a relative discharge of 3.6.

Assuming a mean annual flow at the spill site of  $1.4 \text{ m}^3/\text{s}$ , the relative discharge at the time of the spill is  $2.4/1.4 = 1.7$ . Then by linear interpolation between the relative discharges, it is seen that the traveltime from Apple River to the spill site would be  $5.95 + (14.6 - 5.95) \times (1.7 - 3.6)/(0.82 - 3.6) = 11.9$  hours. Likewise the traveltime from Apple River to Hanover would be 67.1 hours. The traveltime from the spill site to Hanover should, therefore, be  $67.1 - 11.9 = 55.2$  hours.

With the relatively small amount of data shown on Fig. 9, it is possible to estimate the timing of a spill on the river with much better accuracy than would have been possible by use of (12)–(15).

Fig. 10 is a plot of the unit-peak concentrations measured on the Apple River during the two tests. As can be seen from the figure, the unit-peak concentration should be about  $40 \text{ s}^{-1}$  for a traveltime of 55 hours. Recall that the spilled mass was  $50 \text{ kg}$  ( $5 \times 10^7 \text{ mg}$ ) and the mean annual flow rate at Hanover was  $5.0 \text{ m}^3/\text{s}$  so that flow at Hanover is  $(1.7 \times 5.0 \times 1,000) 8,500 \text{ L/s}$ . Assuming a recovery ratio of 1.0, the peak concentration at the intake can be estimated from (4) as

$$C_p = 40 \times 5 \times 10^7 \times 1.0 / (1 \times 10^6 \times 8,500) = 0.235 \text{ mg/L}$$

The time required for the pollution cloud to pass the intake and the river concentration to be reduced to 10% of the peak value ( $0.024 \text{ mg/L}$ ) can be estimated by use of (17) as

$$T_{d10} = 2 \times 10^6 / (40 \times 3,600) = 13.9 \text{ hours}$$

The times for the arrival of the leading edge of the tracer cloud, from Jobson (1996, p. 46), can also be plotted as in Fig. 9. The traveltime of the leading edge of the tracer cloud from the spill site to Hanover can then be estimated using the same procedure as for the peak concentration, as 51.1 hours. After  $51.1 + 13.9 = 65$  hours the pollution cloud should have passed the intake and the concentration reduced to  $0.024 \text{ mg/L}$ .

In conclusion, the pollutant should first arrive at Hanover 51 hours after the spill. The peak concentration should pass the site 55 hours after the spill; and if there are no losses, it should arrive with a peak concentration of  $0.24 \text{ mg/L}$ . By 65 hours after the spill, the concentration should have fallen back to  $0.024 \text{ mg/L}$ . If there are losses or chemical reactions between the spill and the intake, the concentrations will be smaller and a numerical model could be used for predictions.

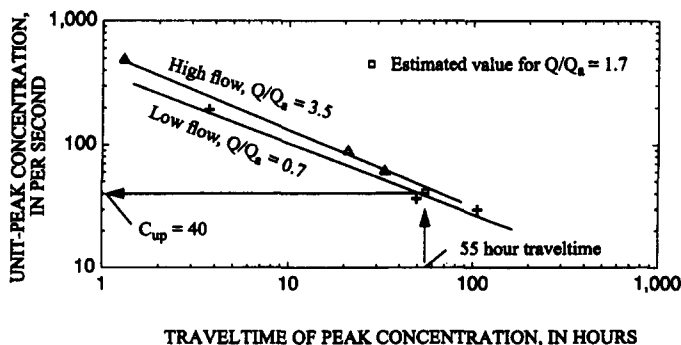


FIG. 10. Unit-Peak Concentrations of Dye for Apple River

## CONCLUSIONS

The possibility of a contaminant being accidentally or intentionally spilled in a river upstream of a water supply is an ever-present danger; and a method of rapidly estimating traveltime and dispersion in rivers is needed. Numerical models are not truly predictive because they must be calibrated with data from the river being modeled. Time-of-travel studies typically provide more accurate traveltime estimates and are much cheaper to conduct than the detailed surveying necessary to obtain adequate channel-geometry data for flow models. Dispersion coefficients cannot be accurately predicted without detailed studies on the river in question.

This paper uses information compiled from a large number of time-of-travel and dispersion studies and presents empirical relations that appear to have general applicability. These relations are not recommended as a substitute for field studies but are believed to provide reasonable estimates in situations where adequate field data are not available. Empirical relations are given for the unit-peak concentrations, velocity of the peak concentration, traveltime of the leading edge of a solute cloud, and the duration of the time of passage as measured from the leading edge to the point where the solute concentration has fallen to 10% of its peak value. This information can be used to estimate the complete response function. The methods are demonstrated by presenting two examples.

If the solute transport in the river is to be modeled, the model must be calibrated to provide the correct traveltimes and rates of attenuation of the peak concentration. The relations presented in this paper can be used to calibrate a solute-transport model for use on a river for which little field data are available.

The relation between unit-peak concentration and traveltime is the best defined of all the relations needed to predict the transport and dispersion of pollutants. The data plotted in Figs. 3–5 show that the peak concentration tends to decrease more rapidly with time than predicted by Fickian dispersion. Because almost all numerical models are based on the Fickian relation, model dispersion coefficients must be assumed to increase with time for the model results to duplicate observed data.

The relation for predicting mean stream velocity (traveltime) is the least accurately defined of all relations presented in this paper. Traveltime information is, therefore, the most valuable information that can be collected to improve the ability to predict the transport and dispersion in a river. These data should be collected at two or more flows, preferably a low flow and a high flow.

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## APPENDIX II. NOTATION

The following symbols are used in this paper:

- $a$  = exponent in the relation of mean stream velocity of discharge;
- $C$  = discharge weighted, cross-sectional average tracer concentration;

- $C_p$  = peak concentration of the tracer cloud;
- $C_u$  = unit concentration (units of inverse time);
- $C_v$  = vertically averaged tracer concentration;
- $C_{up}$  = unit-peak concentration;
- $D_a$  = drainage area of the river at the point of measurement;
- $D'_a$  = dimensionless drainage area based on mean annual discharge;
- $g$  = acceleration of gravity;
- $K$  = constant in the relation of mean stream velocity to discharge;
- $M_i$  = mass of tracer injected;
- $M_r$  = mass of tracer to pass a cross section;
- $n$  = number of sampling site downstream of injection;
- $Q$  = total discharge at the cross section at time  $t$ ;
- $Q_a$  = mean annual flow at the section;
- $Q'_a$  = dimensionless mean discharge;
- $q$  = unit discharge;
- $R^2$  =  $r$  squared value;
- $R_r$  = tracer recovery ratio;
- $S$  = reach slope;
- $T_d$  = duration of the tracer cloud ( $T_r - T_1$ );
- $T_1$  = elapsed time to the arrival of the leading edge of a tracer cloud at a sampling location;
- $T_p$  = elapsed time to the peak concentration of the tracer cloud;
- $T_{pm}$  = minimum probable elapsed time to the peak concentration of the tracer cloud;
- $T_t$  = elapsed time to the trailing edge of the tracer cloud;
- $T_{10d}$  = duration from leading edge until tracer concentration is reduced to within 10% of the peak concentration;
- $t$  = time since injection;
- $V$  = mean stream velocity;
- $V_{mp}$  = probable maximum velocity;
- $V_p$  = velocity of the peak concentration;
- $W$  = total width of the river; and
- $\beta$  = exponent on the unit-peak concentration versus time relation.