SOLAR RADIATION

Important to lakes for two reasons:

1. Provides the energy that controls lake "metabolism" through the conversion of solar energy to chemical energy in the bonds of organic compounds by P/S.
2. Some solar radiation is absorbed or dissipated as heat, which affects the thermal structure and stratification of water masses and circulation patterns of lakes and streams.

ELECTROMAGNETIC SPECTRUM

- Light is received as packets of energy called quanta or photons which have a wavelength (lambda) and an amplitude (A).
- Spectrum ranges from short wavelength, high energy gamma rays (about 100 nm or 1000 Å) to long wavelength, low energy radio and power transmission waves (> 3000 nm or 30,000 Å). Visible spectrum is 400 (violet) to 750 nm (red). Infrared > 750 nm, UV < 400 nm (UV-A 315-400 nm, UV-B 280-315 nm)
- Photosynthetically active radiation (PAR), i.e., radiant energy between 400 and 700 nm wavelengths. Chlorophyll a, the main P/S pigment, has absorption peaks at 445 and 660 nm.
- PAR accounts for about 46-48% of the total energy hitting the earth's surface.
- Solar Constant - amount of direct solar energy per unit time from the sun, incident upon a surface outside the atmosphere perpendicular (at a right angle) to the rays of the sun at an average distance of the earth from the sun. Measured value is about 1.94 $cal/cm^2/min$.

DISTRIBUTION OF LIGHT IMPINGING ON LAKES

1. Reflected - Avg loss is about 5-6% on clear summer day (a good reason to wear sunglasses with UV protection and sunblock when you're on the water). Amount reflected depends on angle of incidence, surface characteristics of water, surrounding topography, and meteorological conditions.
2. Scattered - Some light penetrates the water but is scattered. Scattering is the result of deflection of photons by water molecules, dissolved substances in the water, and suspended particulate matter (dead organic matter, plankton, and nekton). Amount of light scattered can be as much as 25% or more of the amount absorbed.
3. Absorbed - Most of the light entering the water is absorbed, which means photons transfer their energy to the electrons of other atoms and molecules, resulting in heat production and an increase in temperature.

LIGHT PENETRATION IN WATER

- Light Irradiance - measure of the number of photons passing through a unit area ($uE/s/m^2$)
- Light Attenuation - in water there is a rapid reduction in light irradiance with depth due to scattering and absorption of solar energy.
- The most common way of expressing the transmission or absorption of light in water was developed by Birge:

$$\% \text{ Transmission} - 100(I_z/I_0) = 100e^{-k}$$
$$\% \text{ Absorption} - 100^*(I_o-I_z)/I_o = 100(1-e^{-k})$$

where I_o is irradiance at the surface of the lake or some discrete layer within the lake, and I_z is the irradiance at depth z, usually taken at 1-m intervals below I_o and k is the extinction coefficient.

- The absorption of water (in %) is very high in the infrared portion (long wavelengths) and results in rapid heating of water by incident light. Approximately 53% of total light energy is transformed into heat in the first meter of water.
- Absorption by pure water decreases markedly in the shorter wavelengths to a minimum absorption in the blue range and increases again in the violet and ultraviolet (300 nm) wavelengths. Thus, below depths of about 60 m, the only visible light left is usually blue. Dissolved organic carbon (DOC), even low concentrations, greatly increases the absorption of short wavelengths, resulting in more rapid attenuation.
- Light attenuation with depth increases exponentially according to Lambert's Law:

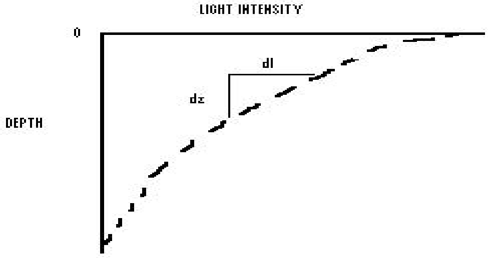
$$I_z = I_o e^{-kz},$$
$$\ln I_o - \ln I_z = kz$$

where k is the extinction coefficient (attenuation coefficient -extinction coefficient; use η in Wetzel) and I_o is the irradiance at the surface and I_z is irradiance at a particular depth z.

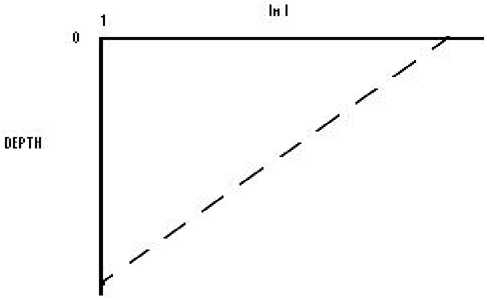
- The extinction coefficient, k, is constant for each wavelength and all wavelengths obey Lambert's Law.
- Total extinction coefficient (k_t) is influenced by water (k_w), absorption of suspended particles in water (k_p), and absorption of dissolved colored substances (k_c):

$$k_t = k_w + k_p + k_c$$

k_t values vary from 0.2 m^{-1} (about 80% transmission) in very clear lakes to 4-10 m^{-1} in highly colored lakes or lakes with high turbidity.



- often converted to a linear plot by taking the log of both sides:
- $\ln Iz = \ln I0 - kz$



components of the attenuation/extinction coefficient

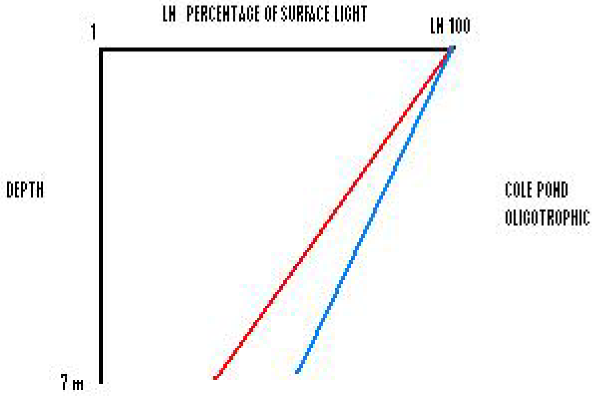
$$K_\lambda = K_{abs} + K_{scattering}$$
$$K = K_{water} + K_{dissolved\ organics} + K_{\ particulates}$$

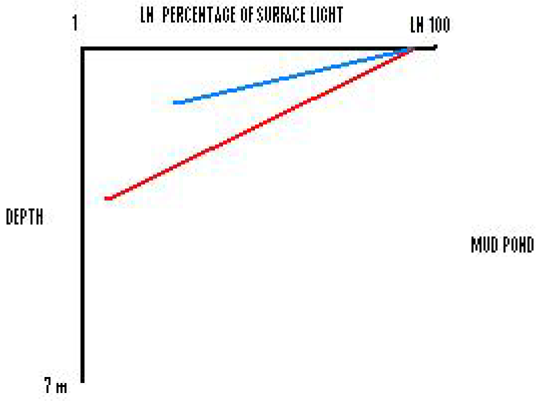
1) **K_{water}**
- for pure water, absorption at long wavelengths dominates (>550 nm; red and IR)
- So, IR disappears in the top 1-2 m of most lakes
- Scattering at short wavelengths, <380 nm
- Pure water does not absorb UV (only scatters it)

- Dissolved salts do not increase attenuation

- 2) Kdissolved organics
  - dissolved organics "Gelbstoff" -- humic and fulvic acids
  - absorb strongly at short wavelengths -- blues and UV's (<500 nm)
- 3) Kparticulates
  - absorbs light evenly over the entire spectrum
  - often the particulates are predominantly tripton and phytoplankton
  - detritus may have higher absorbance at the blue end

Examples from various natural lakes with different amounts of dissolved substances





Modified from Kilham and Likens (1968)

- Different wavelengths of light are attenuated at different rates. In clear water deepest penetration is light in blue region (470 nm). In heavily stained lakes (lots of DOM), no light below 600 nm may penetrate below 1 m depth.
- Of visible light, blue-green penetrates furthest into water; red and violet wavelengths are most rapidly absorbed. Red is absorbed in about the top 1m of water, other wavelengths at different depths until by 60 m only blue remains.
- UV and infrared end of spectrum are absorbed first and so penetrate least distance.
- Pure water order of increasing extinction or decreasing light transmission is Blue, Green, UV, Red, Infrared
- Light is also transmitted through clear ice. If ice is cloudy or covered by snow, attenuation increases greatly.
- Water has color due to upward scatter of light after passing through the water and undergoing differential absorption. Shortest wavelengths are scattered by H_2O molecules the most, with effect proportional to $(1/\text{wavelength})^4$; blue light is thus strongly back scattered to surface by water molecules producing characteristic blue hue.

- Several color scales have been developed to measure true color after suspended substances have been filtered out. Most common color scale in North America is cobalt-platinum scale. Ranges from 0 platinum units in very clear lakes to 300 Pt units in heavily stained bog waters, which have high concentrations of humic substances.

TRANSPARENCY

Depth to which visible light penetrates or transparency is measured in 3 ways:

1. Underwater photometers which use flat sensor (some have 4 pi geometry, circular)
2. Scanning spectroradiometers
3. Secchi disc depth (z_{sd}) - Secchi disc is a metal disc 20 cm in diameter divided into 4 quandrants on its upper face, two of which are white and two of which are black.

- Secchi disc depth (transparency) is the mean depth at which the Secchi disc disappears when viewed from the shaded side of a boat and at which it reappears when raised after it has been lowered beyond visibility.
- z_{sd} correlates reasonably well with % transmission and on average it corresponds to about 10% of surface light (ranges from 1-15%)
- During the ice-free period z_{sd} is strongly related to extinction coefficient by:

$$k\ (m^{-1}) = 1.7/z_{sd}\text{, in lakes, and}$$
$$k\ (m^{-1}) = 1.45/z_{sd}\text{, in oceans}$$

- z_{sd} is a function of the absorption characteristics of water, the concentration of dissolved organic matter (DOM) and the concentration of particulate organic matter (POM). Measured values range from a few cm in highly turbid waters to > 40 m in ultra clear water. Most measurements in Ontario range from 2-10 m in depth.
- Seasonal variations in transparency related to variations in phytoplankton abundance or inorganic particle concentration are reflected by variations in z_{sd}.

LIGHT ZONATION IN LAKES

Because of the importance of light for P/S it imposes structure on lakes, both vertically and horizontally.

1. PHOTIC or Euphotic Zone - extends from surface to depth where light is 1% of incident light at surface. Region of net O_2 production during the day due to photosynthesis (P/S) O_2 declines during night - respiration (decomposition processes)
2. APHOTIC Zone - extends from bottom of photic zone to bottom of lake. Light levels too low for photosynthesis (P/S) so respiration processes dominate over production processes and aphotic zone uses O_2.

- Lower boundary of photic zone varies daily and seasonally with changing light intensity and water transparency
- Compensation Depth - depth at which incident light is reduced to about 1% so P/S = respiration.
- Oligotrophic lakes - unproductive lakes, photic zone may be 20-25 m deep; e.g., offshore waters of Lake Superior 15-20 m.
- Eutrophic Lakes - highly productive, photic zone may be < 1 m deep.
- In most cases P/S efficiency of phytoplankton becomes limited at light levels less than 10% of surface irradiance. Thus the effective photic zone may be shallower than one based on the compensation depth.

TEMPERATURE

HEAT

Two functions in lakes:

1. Thermal stratification (temperature zonation). Temperature is a measure of the intensity, not the quantity, of heat in a waterbody
2. Regulates rates of chemical and biological processes

SOURCES:

1. Direct solar radiation
2. Conduction from the atmosphere
3. Conduction from the bottom sediments
4. Condensation of water vapour at the surface
5. From terrestrial sources via runoff and groundwater

LOSSES:

1. Radiative losses to the air at the surface and to much lesser extent to sediments
2. Evaporation
3. Outflows, especially streams

TEMPERATURE ZONATION IN TEMPERATE LAKES

Water column of temperate lakes has three layers during summer (thermal stratification)

- Epilimnion - warm upper layer
- Metalimnion - layer of rapid change in temperature (e.g., thermocline)
- Hypolimnion - cold bottom layer, about 4 C
- Structure of water column due to temperature reflects differences in water density.
- Greater density change per degree of temperature change in warm water than in cold, e.g., takes about 30X as much energy to mix equal volumes of 24 & 25 C water completely as it takes to mix same volumes of 4 & 5 C water.
- All or part of epilimnion may be stirred well during summer. Mixed layer is water that is well mixed by wind. Mixed layer may be very deep or shallow depending on season and interactions between sun and wind on a day. In summer all or part of epilimnion may be mixed. In fall or spring entire water column is mixed top to bottom. Ice cover prevents mixing in winter.
- Temperate lakes thermocline spans 10-15 C range. Tropical lakes, due to unique temperature-density relationship of water, stable stratification occurs with a thermocline spanning only 1-3 C
- THERMOCLINE - historically defined as region where temperature changes are > 1 C per m depth. This works in temperate lakes but not tropical lakes where temperature differences during stratification may only be 1-3 C. In practice, region of temperature change (i.e., metalimnion) is often referred to as the thermocline.
- Deepest or PARENT or SEASONAL thermocline always lies within metalimnion
- The resistance of a lake to mixing after thermal stratification is a measure of its mechanical stability. We can estimate the amount of work (in ergs) required to mix the lake completely based on temperature-density relationships, but usually it is more convenient to make comparisons rather than absolute measurements between successive layers of a water column. Usually these comparisons are made against the difference in density of water at 4 and 5 C, (8.1×10^{-6} g/cm^3)

Relative Thermal Resistance = $[(p_2-p_1)/8.1] \times 10^6$
where p_2 and p_1 are densities at adjacent temperatures at the bottom and top respectively, of a column of water. Relative thermal resistance is always a positive value.

THERMAL STRATIFICATION

1. SPRING OVERTURN (MIXING) - Sun heats water surface. Wind stirs warm surface water, which is less dense, down to depth where turbulence eventually dissipates. This depth becomes top of thermocline. Because this down mixing water is warmer and positively buoyant it resists mixing in proportion to the density difference. More heat is absorbed in first few meters of water and to extend further down in water column, it must be physically stirred by wind or convection-induced turbulence. Density of water changes rapidly with temperature so a large effect can be expected with a few days of sun and calm weather.
2. SUMMER STRATIFICATION - Once spring thermocline is established, it is thermodynamically stable and can be destroyed only by cooling of the epilimnion. Hurricane strength winds will sharpen boundaries between the water layers but they will not cause a lake to destratify. Hypolimnion is effectively isolated from the surface. Dissolved oxygen cannot be replenished except by diffusion from the metalimnion which is very slow. Once a lake is stratified, direct heating is the only important source of heat to the hypolimnion. In some lakes geothermal heating may occur . Direct heating occurs when water is sufficiently transparent to allow light to penetrate below the thermocline.
3. FALL OVERTURN - In the fall less solar radiation reaches lake surface during day and heat losses are greater at night. Cooling water is denser than warmer water below and so it sinks forcing warm water up to the surface. These convective currents and wind mixing begin to weaken the thermocline. Epilimnion increases in depth as temperature decreases. Eventually temperature and density differences between adjacent layers is so slight that strong wind overcomes remaining resistance to mixing and lake undergoes fall overturn and mixes from top to bottom.
4. WINTER STAGNATION - After the fall overturn the lake is homoiothermous and mixed top to bottom. This mixing continues until surface freezes. Ice cover prevents mixing by winds. Freezing occurs when surface waters reach 0 C on a windless, cold night. Some slightly warmer water (4 C) remains below the ice producing an inverse stratification within a few cm of the bottom of the ice. Clear ice with no snow cover allows solar radiation to penetrate into the water column so some heating may occur during the winter.

THERMAL BAR

- Thermocline does not form all over lake at same time in large cold lakes such as the Great Lakes. Thermal bars do not usually form in small lakes because winds are sufficiently strong to mix nearshore and offshore waters.
- During spring waters of the Great Lakes become divided into zones: offshore (i.e., pelagic zone) unstratified water at less than 4 C and warmer weakly stratified mass near shore (e.g., littoral zone) > 4 C. 4 C water between the two zones is densest and sinks; this zone of dense sinking water is the thermal bar.
- Near shore water is warmer because it is relatively shallow so heat is contained in a small volume.
- Thermal bar is a vertical rather than horizontal temperature discontinuity.
- Thermal bar gradually moves offshore as heating of inshore areas continues.
- Thermal bar enhances early algal growth by effectively trapping heat. Nutrients and toxics are also trapped within the thermal bar. Thermal bars also occur at a time of year when many organisms are spawning and reproducing. This means eggs, embryos and young larvae are potentially exposed to less than ideal developmental conditions.

THERMAL LAKE TYPES

Based on the occurrence of mixing, how much of the water column mixes, number of mixing events per year, and temperature at the time of mixing.

A. HOLOMICTIC - lake that mixes from top to bottom during annual mixing cycle. Among holomictic lakes there are 5 types based on the frequency of mixing.

1. DIMICTIC - lakes that mix twice per year, once in fall before ice-up but after turnover and once in spring after ice breaks up and before stratification begins. Ice cover in winter prevents mixing.
2. MONOMICTIC - lakes that are never completely ice covered; have one long mixing period throughout the winter; e.g., Great Lakes except Erie. Cold monomictic lakes temperature never > 4 C. Mostly arctic and mountainous regions. Warm monomictic lakes temperature never < 4 C. Found in warmer parts of temperate zone and mountainous regions of the subtropics.
3. POLYMICTIC - lakes that are shallow and mix every few days or even daily all year round. Warm polymictic lakes temperature >4 C all year, generally tropical. Cold polymictic lakes temperature is approximately 4 C, found in high mountainous areas in equatorial regions where seasonal changes in air temperature are small. Warm polymictic lakes have temperatures well above 4 C during periods of circulation.
4. AMICTIC - lakes that have year round ice cover and never mix.
5. OLIGOMICTIC - lakes that thaw once every few years and mix, generally in Arctic areas. In some cases warm lakes in which temperatures always higher than 4 C undergo circulation at irregular intervals. Stratification is broken by exceptionally cool weather. Found at low elevations in tropical areas.

B. MEROMICTIC - Deep or chemically stratified lakes that mix only partially because there is insufficient energy to overcome stratification and stir it top to bottom.

- Lakes with significant accumulations of dense salty water near bottom may not mix. Such lakes are chemically meromictic and have a warm bottom layer or MONIMOLIMNION, where buoyancy of deep warm water is counterbalanced by increased density of dissolved salts.
- The upper layer that does mix is called the MIXOLIMNION. Mixing of meromictic lakes by severe storms is dramatic because H_2S accumulated in the anoxic bottom waters reaches surface, O_2 is depleted and massive fish kills etc. occur. Nutrients released usually produce a series of algal blooms until equilibrium is reestablished.

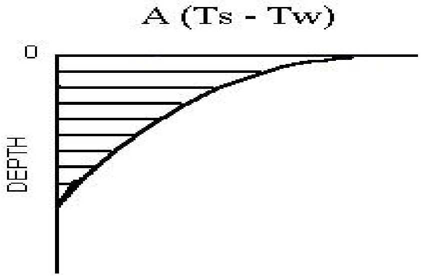
HEAT BUDGETS

Measure of heat storage capacity of a lake.

1. Annual heat budget - the total amount of heat necessary to raise water from the minimum temperature in winter to the maximum temperature in summer.
2. Summer Heat Income - amount of heat needed to raise the temperature of the lake from the homothermal condition of 4 C in spring to the maximum observed summer temperature.
3. Winter Heat Income - amount of heat needed to raise the temperature from its minimum heat content (i.e., lowest winter temperature) to 4 C.

Annual heat budget increases with mean depth (z), area (A_o), and volume (V).

A = Area at depth z
Ts = maximum summer temperature at depth z
Tw = minimum winter temperature at depth z



integrate

Analytical heat budget – budget based on identification of all the sources and sinks for heat to or from a lake

HYDRODYNAMICS

- Wind, solar radiation, gravity, and earth's rotation (Coriolis force) are important forces causing water movements in lakes.
- Within a lake morphometry of the basin, stratification structure (density gradient), and exposure to wind (fetch) are important considerations.
- Knowing water movements is important because currents and waves influence the distribution of dissolved substances (e.g., gases), nutrients and food, and the distribution of microorganisms and plankton.
- Surface currents - nonperiodic, net unidirectional water flows. Normally exhibit sheared flow, which means velocity decreases with depth due to drag imparted by viscosity.
- Waves - periodic or rhythmic water flows.

Water Flow

- Water flows can either be laminar or turbulent. Laminar flow occurs at slow speeds and is characterized by smooth, unidirectional movement (i.e., ordered) with a uniform velocity profile. Turbulent flow occurs at higher speeds and is disordered, chaotic, and multidirectional with sheared flow profiles (reduction in flow with depth resulting from drag imparted by viscosity), that ultimately results in eddy formation.
- Shift from laminar to turbulent flow in a fluid in a smooth tube is related to the viscosity and density of the fluid, its velocity, and the size of the channel or tube through which the fluid flows.
- Reynolds Number, R_e, is used to determine if flow is laminar or turbulent in a tube.

$$\mathbf{R_e = (pDv)/\mu,}$$

p is density, D is the diameter of the channel or tube, v is velocity, and μ is viscosity.

- $R_e < 1{,}000$ flow is laminar
- $R_e > 1{,}000$ flow is turbulent.
- Using depth for D in lakes, velocities of only a few mm/s will induce turbulent flow, with $R_e \sim 10^6$. LAMINAR FLOW IS ALMOST NEVER OBSERVED IN THE EPILIMNION OF STRATIFIED LAKES or in UNSTRATIFIED WATER.
- At epilimnion-metalimnion and metalimnion-hypolimnion interfaces, there is usually laminar flow. Water in each layer flows in opposite directions.

- In stratified lakes two important, but opposing forces are working:
  1. the vertical velocity gradient (called vertical shear) which forces mixing, and
  2. the vertical density gradient which prevents mixing of adjacent layers.
- Whether or not mixing occurs between these miscible layers, i.e., the stability of stratification, can be predicted using the Richardson number, R_i. The Richardson number is the ratio b/a:

$$\mathbf{R_i = [g(dp/dz)]/[p(dv/dz)^2]}$$

g is acceleration due to gravity (9.8 m/s^2), is density, v is velocity, and z is depth.

- $R_i < 0.25$ in stratified fluid subjected to shearing flow, increases in friction and in mixing perpendicular to current direction occur, i.e., stratification is not stable.
- $R_i > 0.25$, flow remains stable and stratification is stable. There is no mixing of adjacent layers as they flow by one another because friction is low.
- When $R_i < 0.25$, the critical velocity difference along a the density interface (where two layers of different densities are in contact with each other) is exceeded, so disturbances grow in amplitude and vortices form.
- Vortices increase mixing since they form a transitional layer in which there is both shear (velocity gradient) and a density gradient.
- If vortex formation and mixing occurs, the transitional layer will occur in the epilimnion since there is usually no density gradient. The metalimnion is generally stable since there is rarely substantial water flow, except when internal seiches occur.

EDDIES

- Eddies are parcels of water with circular motion (swirls). Technical definition - an assemblage of shear waves of a spectrum of many lengths or "eddy diameters". Eddies have both vertical and horizontal motion and increase the diffusion of heat and dissolved substances (turbulent diffusion).
- Turbulence consists of a series of nested eddies of varying sizes in which energy is transferred from largest to progressively smaller eddies until viscosity prevents further eddy formation.
- The rate of turbulent diffusion is a function of the eddy diffusion coefficient (K_z), which is a measure of the rate of exchange or intensity of mixing across a plane. K_z varies with average density, velocity of vertical motion, and mixing length.
- K_z varies inversely with stability. Thus K_z decreases from turbulent epilimnion into more stable metalimnion and hypolimnion.

Surface Waves

- Wind Drift - surface motion produced by friction of wind blowing over water surface.
- Traveling Surface Waves (Progressive waves) - occur when wind sets surface into oscillation. These are generally of short wavelength and confined to surface layers. Limnological significance is minimal except in shore areas. Causes surface water particles to move in a circular orbit called cycloid. Vertical motion of cycloids decreases with increasing depth, cycloid diameter halved for every depth increase of wavelength/9.
- Wavelengths of short surface waves are less than water depth (except in shore areas) so they travel at speeds proportional to wavelength$^{1/2}$. These waves are also called deepwater waves and are said to be dispersive.
- In moderate to large lakes maximum height of short waves (H) is related to fetch (X)

$$\mathbf{H\ (cm) = 0.105 * sqrt\ (X, cm)}$$

e.g., based on a fetch of 482 km for Lake Superior, the predicted wave height is 7.3 m, which is consistent with the maximum observed height of 6.9 m.

- Shallow water or long waves have wavelengths > 20 X water depth and speed is proportion to depth$^{1/2}$. Because speed is not related to wavelength, these waves are not dispersive. Cycloid motion of water is transformed into back and forth movements which extend to the bottom of the water column.
- As shallow water waves enter shallow areas, height increases until it collapses over the front as a breaker.

WHOLE LAKE MOVEMENTS

- Generally waves with wavelengths similar to basin dimensions and can be either oscillations of the water surface or isotherm depth (thermocline).
- Long waves are reflected by the basin boundaries and produce standing waves characterized by nodes about which substantial horizontal motion occurs while vertical motion is nil.
- These standing waves are **SEICHES**, from French meaning to dry, referring to periodic drying exposures of shallow littoral zones.
- Seiches are most often caused by wind induced tilting of the water surface and thermocline. When wind stops lake flows back towards equilibrium but overshoots. These oscillations continue but are dampened each time by friction and gravity.

Surface Seiches

- Occurs in unstratified lakes. Maximum amplitude occurs on the surface but it involves the entire water column.
- Surface seiches are generated when winds blow fairly constantly in one direction, driving surface water downwind. Water piles up on lee shore and wind holds it there until it drops, at which time driving force is released and accumulated water mass flows back under gravity. Produces standing wave that rocks back and forth with decreasing motion which is damped by friction and gravity.
- Surface seiches are apparent on lakes as small up and down motions.
- Period of oscillation at the node, t is

t = (2l)/(sqrt(g*zbar))

where t is the period (sec), l is length of basin at surface (cm), g is acceleration due to gravity (9.8 m/s^2), and zbar is mean depth.

- Unimodal surface seiches are common. Multimodal seiches can be generated by periodic exertion and release of pressure on the surface in the centre of a basin.
- Amplitude is usually small compared to internal seiches and impacts are variable. Generally not important to chemistry or biology of lake as energy is low. For example, in Lake Erie surface seiches can exceed 2 m (t=14 hr) and can have positive and negative effects, flushing of river deltas and harbour areas, shoreline property damage and erosion.
- Surface seiches can occur under ice cover as well. They are common in Lake St. Clair during spring ice breakup.

Internal Seiches

- Occur when lake is stratified and involve different density layers oscillating in a standing wave relative to one another. Most frequently occur at the thermocline and are usually detected by the rise and fall of the thermocline. Occur only under stratified conditions and aren't apparent from surface. Usually much larger than surface seiches and may be as high as 10 m and period is often much longer. Small lakes usually not affect by internal seiches.

- Results from large scale horizontal water movements due to wind. Resulting currents flow rhythmically back and forth in opposite directions and constitute major deepwater movement in lakes.
- Internal seiches are important because transport heat and dissolved substances large distances both vertically and horizontally and they may significantly alter the distribution and productivity of phytoplankton (algae) and zooplankton, directly or indirectly through changes in thermal and chemical stratification.

- Motion of seiches is not simply back and forth movement since they are subject to Coriolis force. This results in a circular track called an inertial circle with flow deflected onto the right shore in the Northern Hemisphere.

Resistance to mixing and stability

- Resistance to mixing proportional to $d\rho / dz$

- Stability – the resistance to mixing; the amount of work that would be required to mix an entire lake to uniform density without adding or subtracting heat in the process.

1. Whole lake stability – determines if the whole lake will mix; is the amount of energy required to mix the entire lake to uniform density (KJ/cm3)

a. $S = 1/A_0 \sum_{Z_0}^{z_{max}} (\rho_z - \rho_{average}) (z - z_{\rho average}) (A_z)\, dz$

where Σ is actually an integral symbol

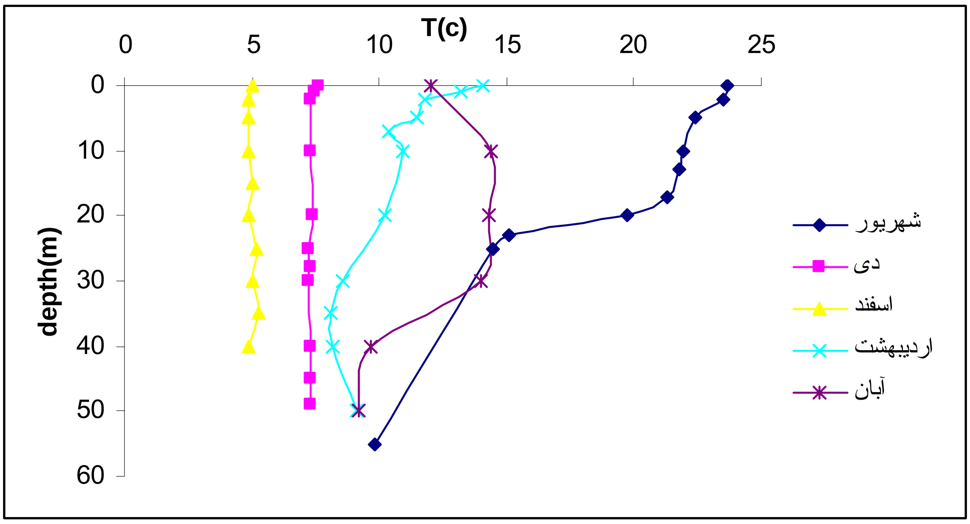
where A_o = the surface area in cm
A_z = the area at some depth z (in cm)
$\rho_{average}$ = the final or mean density that would result if the lake were completely mixed
ρ_z = the density at depth z
$z_{\rho average}$ = the depth (cm) where the final (mixed) mean density exists prior to mixing
z_{max} = maximum depth in cm
z_0 = surface or zero depth

2. **Richardson's stability** – determines whether or not two fluids will mix

a. $Ri = (g \times d\rho/dz) / \rho_{average} (du/dz)^2$
Where g = acceleration of gravity
ρ = density
u = horizontal velocity

b. $Ri > 0.25$ then no mixing -- numerator dominates

c. $Ri < 0.25$ then will mix -- denominator dominates (energy of mixing)



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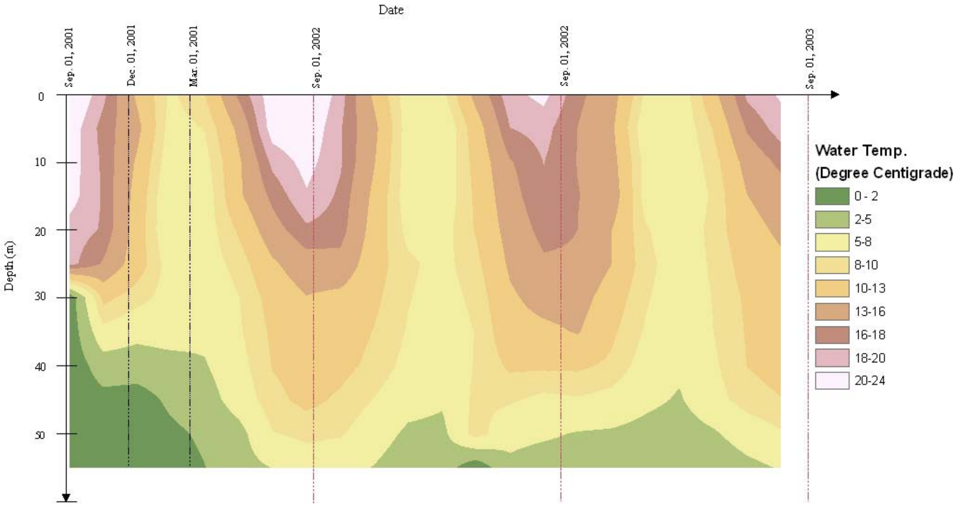

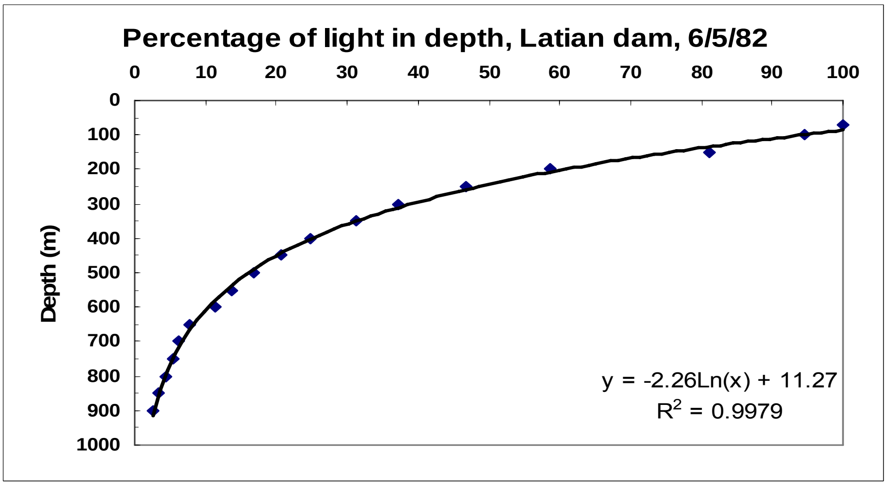

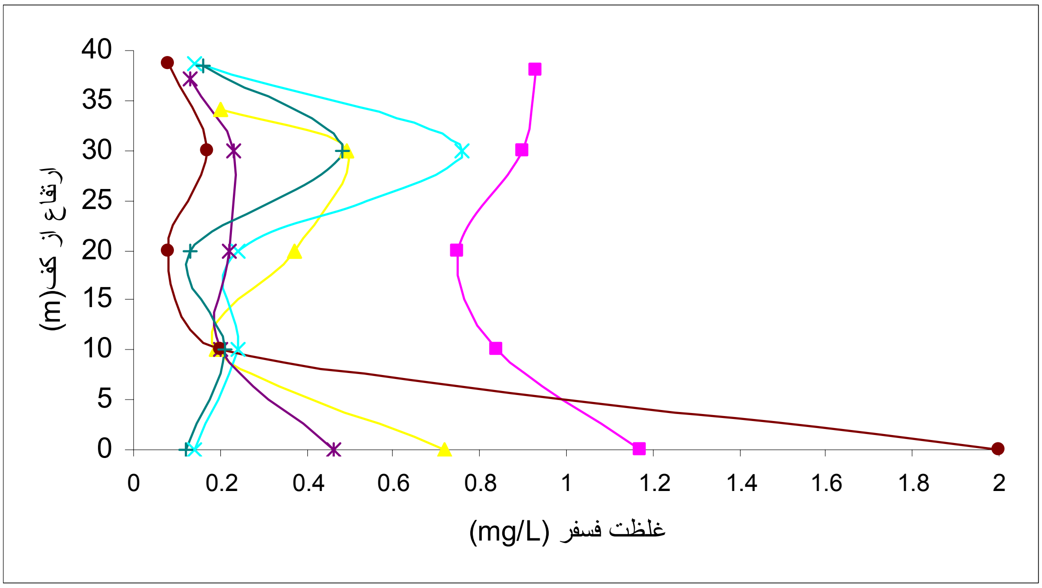
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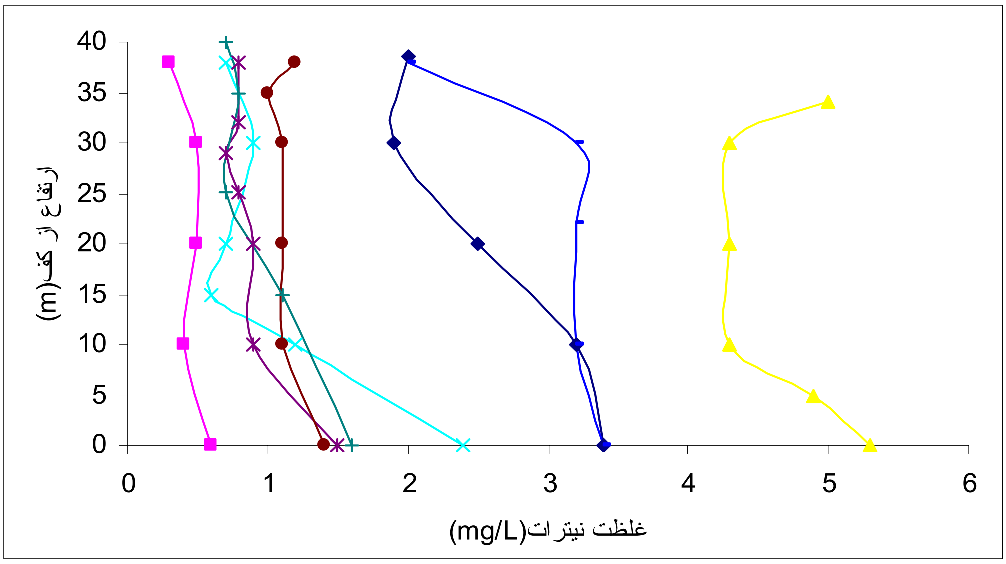
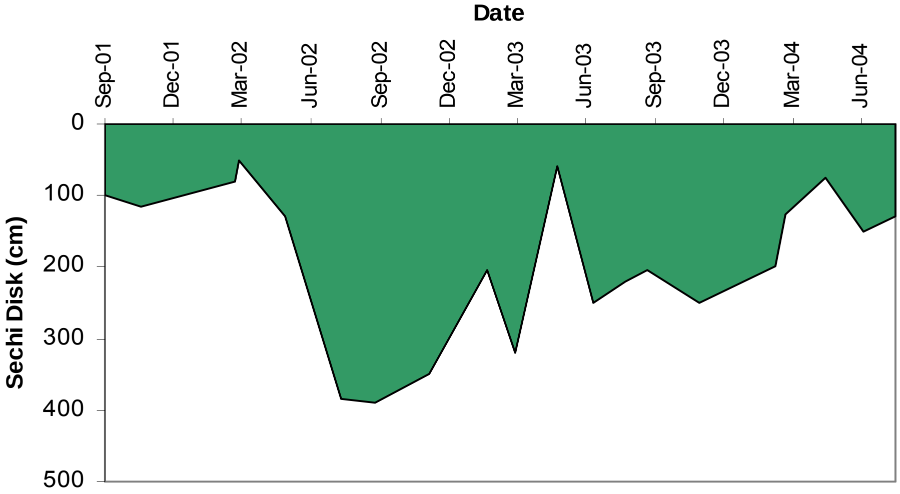
2
غلظت فسفر (mg/L)


ارتفاع از کف(m)
40
35
30
25
20
15
10
5
0
0
1
2
3
4
5
6
غلظت نیترات(mg/L)


Date
Sep-01
Dec-01
Mar-02
Jun-02
Sep-02
Dec-02
Mar-03
Jun-03
Sep-03
Dec-03
Mar-04
Jun-04
Sechi Disk (cm)
0
100
200
300
400
500