

20.11. The epilimnion of a lake ($H = 7$ m) has a temperature of 16°C and is subject to a constant wind of 2 mps. (a) Estimate the oxygen reaeration rate in d^{-1} . (b) Estimate the mass transfer velocity for ammonia gas in m d^{-1} . Assume Eq. 20.46 holds.

20.12. A wide river ($B = 200$ m) has a flow of 800 cms and a mean depth of 2.7 m. It is subject to a wind of 1.5 mps and its mean temperature is 25°C .

(a) Determine its reaeration rate in d^{-1} .

(b) Determine a mass-transfer rate for ammonia in m d^{-1} . Assume Eq. 20.47 holds.

LECTURE 21

Streeter-Phelps: Point Sources

LECTURE OVERVIEW: I derive the classic Streeter-Phelps equation for a single point source of carbonaceous BOD and provide some guidance on how it can be calibrated. Next I show how it can be used to model multiple point sources and illustrate how care must be taken when making an oxygen balance for a point source. I then show how the model is affected by anaerobic conditions. Finally I contrast the plug-flow model with the mixed-flow version that is appropriate in systems such as estuaries, where dispersion is significant.

The Streeter-Phelps model ties together the two primary mechanisms governing dissolved oxygen in a stream receiving sewage: decomposition of organic matter and oxygen reaeration. As such it provides an analytical framework for predicting the effect of both point and nonpoint sources of organic wastewater on stream and estuary dissolved oxygen. In this lecture I focus on point sources.

21.1 EXPERIMENT

Now that we have explored ways to characterize gas transfer, let us return to the experiment that we developed in Lec. 19. Recall that we placed a quantity of organic matter in a sealed bottle and then simulated how oxygen was depleted. Now we can perform the same simulation, but with the bottle open to the atmosphere.

The mass balances for BOD and dissolved oxygen can be written as

$$V \frac{dL}{dt} = -k_d VL \quad (21.1)$$

$$\text{and } V \frac{dO}{dt} = -k_d VL + k_a V(O_s - O) \quad (21.2)$$

Now before proceeding with the solution, we make a transformation that simplifies the oxygen balance. To do this we introduce a new variable,

$$D = O_s - O \quad (21.3)$$

where D is called the *dissolved oxygen deficit*. Equation 21.3 can be differentiated to give

$$\frac{dD}{dt} = -\frac{dO}{dt} \quad (21.4)$$

Equations 21.3 and 21.4 can be substituted into Eq. 21.2 to give

$$V \frac{dD}{dt} = k_d VL - k_a VD \quad (21.5)$$

Thus the use of deficit simplifies the differential equation.

If $L = L_0$ and $D = 0$ at $t = 0$, then Eqs. 21.1 and 21.5 can be solved for

$$L = L_0 e^{-k_d t} \quad (21.6)$$

$$\text{and } D = \frac{k_d L_0}{k_a - k_d} \left(e^{-k_d t} - e^{-k_a t} \right) \quad (21.7)$$

As depicted in Fig. 21.1, opening the lid means that oxygen first decreases, but it then recovers as reaeration replenishes the oxygen.

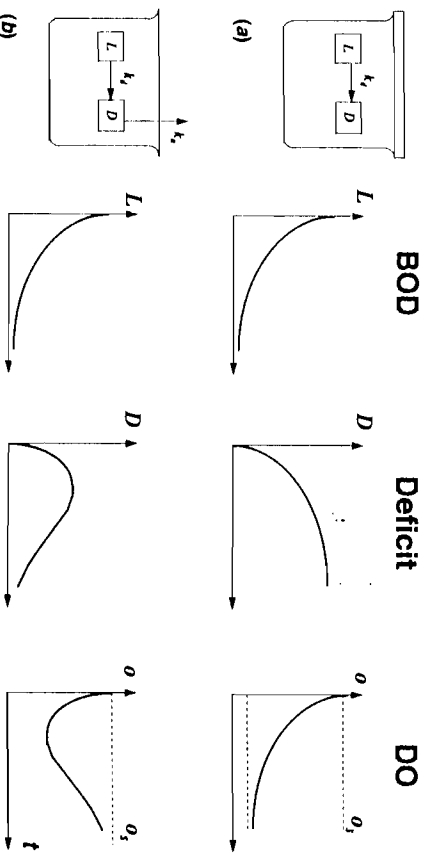


FIGURE 21.1 How (a) closed and (b) open systems are affected by BOD decomposition.

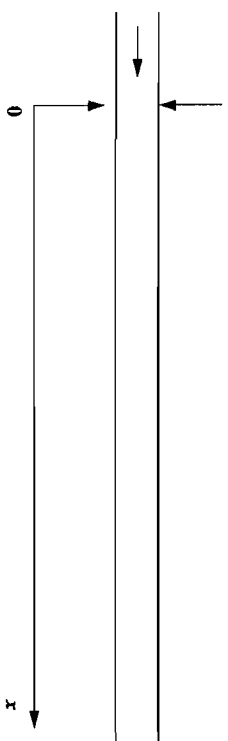


FIGURE 21.2 A point source discharged into a stream reach of constant geometry and hydrology.

21.2 POINT-SOURCE STREETER-PHELPS EQUATION

Now we can move the foregoing theory into a natural water. Specifically we will model a stream with a single point source of BOD. As depicted in Fig. 21.2 the reach is at steady-state and is characterized by plug flow with constant hydrology and geometry. This is the simplest manifestation of the classic Streeter-Phelps model.

Mass balances can be written as

$$0 = -U \frac{dL}{dx} - k_r L \quad (21.8)$$

$$\text{and } 0 = -U \frac{dD}{dx} + k_d L - k_a D \quad (21.9)$$

where $k_r = k_d + k_s$.

If $L = L_0$ and $D = D_0$ at $t = 0$, then these equations can be solved for

$$L = L_0 e^{-\frac{k_r}{U} x} \quad (21.10)$$

$$\text{and } D = D_0 e^{-\frac{k_a}{U} x} + \frac{k_d L_0}{k_a - k_r} \left(e^{-\frac{k_r}{U} x} - e^{-\frac{k_a}{U} x} \right) \quad (21.11)$$

These equations constitute the classic "Streeter-Phelps" model and are the equations behind the oxygen "sag" we discussed in Fig. 19.2.

Aside from being written for a plug-flow system, these equations differ from Eqs. 21.6 and 21.7 in two important ways. First, only part of the removal has an effect on the deficit. Second, the deficit has an initial value.

21.3 DEFICIT BALANCE AT THE DISCHARGE POINT

We introduced the dissolved oxygen deficit to simplify our mathematical model. Unfortunately it also complicates the determination of the boundary concentration at the mixing point because most waste inflows exhibit a significantly different temperature from the receiving water. Consequently their oxygen saturations will differ and, hence, a simple deficit balance will be erroneous. This is best illustrated by example.

EXAMPLE 21.1. OXYGEN BALANCE AT A DISCHARGE POINT. A point source and a receiving stream at sea level have the following characteristics:

Values	Point source	River
Flow ($\text{m}^3 \text{s}^{-1}$)	0.463	5.787
Temperature ($^{\circ}\text{C}$)	28	20
DO (mg L^{-1})	2	7.5
DO saturation (mg L^{-1})	7.827	9.092
DO deficit (mg L^{-1})	5.827	1.592

Perform mass balances for temperature and oxygen assuming complete mixing.

Solution: If we assume that the density and heat capacity of water are relatively constant, a heat balance for temperature can be developed in a similar fashion to a mass balance,

$$T_0 = \frac{5.787(20) + 0.463(28)}{5.787 + 0.463} = 20.59^{\circ}\text{C}$$

A mass balance for oxygen can be calculated simply as

$$o_0 = \frac{5.787(7.5) + 0.463(2)}{5.787 + 0.463} = 7.093 \text{ mg L}^{-1}$$

The saturation value for 20.59°C is 8.987 mg L^{-1} . Therefore the deficit at the mixing point can be determined as $D_0 = 8.987 - 7.093 = 1.894 \text{ mg L}^{-1}$. This is the correct value.

Now let us see what happens when we try to balance the deficit directly:

$$D_0 = \frac{5.787(1.592) + 0.463(5.827)}{5.787 + 0.463} = 1.906 \text{ mg L}^{-1}$$

which represents a discrepancy of $1.894 - 1.906 = -0.012 \text{ mg L}^{-1}$. Why does this error occur? It results from the fact that temperature and oxygen saturation are related in a nonlinear fashion by Eq. 19.32.

It should also be mentioned that the type of errors illustrated in the previous section can also be incurred for systems where saturation changes longitudinally due to temperature, elevation, or salinity variations. Examples include systems with high elevation changes (upland streams) or estuaries. For such systems, deficit balances at junctions can introduce slight discrepancies if not handled correctly. Several end-of-chapter problems illustrate the errors.

Fortunately the discrepancy incurred by using deficit is usually not great because the relationship of oxygen to temperature, elevation, and salinity is not highly nonlinear over the ranges commonly encountered (recall Figs. 19.9 and 19.10). Therefore the errors incurred are usually not significant. In addition, now that computers are ubiquitous, dissolved oxygen (rather than deficit) is simulated directly, and the problems associated with deficit balances are becoming a moot point.

21.4 MULTIPLE POINT SOURCES

Now that we have an understanding of how to perform mass balances at discharge points, we can investigate how multiple point sources can be simulated. In general the approach consists of treating the river as a series of uniform reaches linked by boundary conditions. The computation starts at the farthest upstream point where boundary concentrations are specified. Then the model equations (Eqs. 21.10 and 21.11) are used to calculate concentrations in the downstream direction.

The calculation is continued until a boundary is encountered. Two types of boundaries occur. The first represents the case where the system parameters change. For example the bottom slope might change, which would lead to a change in the channel's velocity, depth, reaeration rate, etc. In such cases the concentrations at the end of the upstream reach directly serve as the initial concentrations for the model equations with revised parameters. Second, for situations where a point source enters the system, mass balances are used to establish the starting concentrations for the new reach. The approach is described in the following example.

EXAMPLE 21.2. MULTIPLE SOURCES. Figure 21.3 shows a river that receives a sewage treatment plant effluent at kilometer point 100 (KP 100) and a tributary inflow at KP 60. Note that the channel is trapezoidal with the characteristics shown. The deoxygenation rate for CBOD is equal to 0.5 d^{-1} at 20°C . For 20 km downstream from the treatment plant, there is a CBOD settling removal rate of 0.25 d^{-1} .

Assuming that the O'Connor-Dobbins reaeration formula holds and that the stream is at sea level, compute the concentration of dissolved oxygen in the system. To simplify the calculation, we have independently determined the heat balances. In addition we have computed the system's hydrogeometric parameters and reaction kinetics. This information is included in the following tables:

Parameter	Units	KP > 100	KP 100-60	KP 80-60	KP < 60
Depth	m	1.19	1.24	1.24	1.41
	(ft)	(3.90)	(4.07)	(4.07)	(4.62)
Area	m^2	14.71	15.5	15.5	18.05
Flow	$\text{m}^3 \text{ s}^{-1}$	5.787	6.250	6.250	7.407
	$\text{m}^3 \text{ d}^{-1}$	500.000	540.000	540.000	640.000
	(cfs)	(204)	(221)	(221)	(262)
Velocity	m s^{-1}	0.393	0.403	0.403	0.410
	m d^{-1}	33.955	34.819	34.819	35.424
	(fps)	(1.29)	(1.32)	(1.32)	(1.35)

Parameter	KP > 100	KP 100-80	KP 80-60	KP < 60
T ($^{\circ}\text{C}$)	20	20.59	20.59	19.72
o_s (mg L^{-1})	9.092	8.987	8.987	9.143
k_d (d^{-1})	1.902	1.842	1.842	1.494
k_r (d^{-1})	0.50	0.764	0.514	0.494
k_t (d^{-1})	0.50	0.514	0.514	0.494

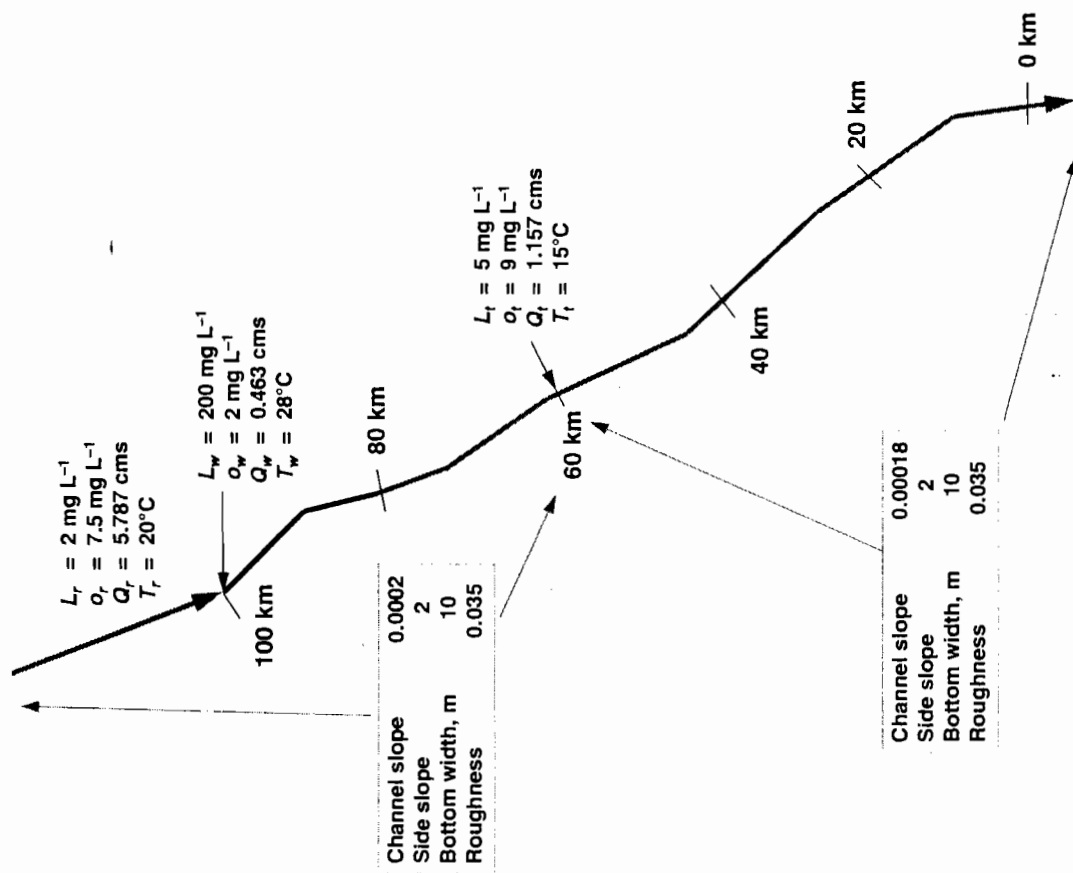


FIGURE 21.3
A stretch of stream with two point sources and changing hydrogeometric characteristics.
Solution: First, the concentration of carbonaceous BOD can be calculated for each stretch. A CBOD mass balance is developed at KP 100,

$$L_0 = \frac{40,000(200) + 500,000(2)}{540,000} = 16.667 \text{ mg L}^{-1}$$

which then decays downstream according to

$$L = 16.667e^{-\frac{0.514+0.25}{34.819}x}$$

At KP 80 (that is, 20 km downstream from the point source) the value of BOD will have dropped to

$$L = 16.667e^{-\frac{0.764}{34.819} \cdot 20,000} = 10.75 \text{ mg L}^{-1}$$

In the next reach the BOD does not settle, but continues to decay. At KP 60 the value of BOD will have dropped to

$$L = 10.75e^{-\frac{0.514}{34.819}(40,000-20,000)} = 8 \text{ mg L}^{-1}$$

Now another mass balance must be taken to account for the inflowing tributary,

$$L_0 = \frac{540,000(8) + 100,000(5)}{640,000} = 7.53 \text{ mg L}^{-1}$$

which then decays downstream. Thus by KP 0, the value of BOD will have dropped to

$$L = 7.53e^{-\frac{0.494}{35.424}(100,000-40,000)} = 3.26 \text{ mg L}^{-1}$$

For dissolved oxygen a mass balance is developed at KP 100,

$$o_0 = \frac{40,000(2) + 500,000(7.5)}{540,000} = 7.093 \text{ mg L}^{-1}$$

which represents an initial deficit of

$$D_0 = 8.987 - 7.093 = 1.894 \text{ mg L}^{-1}$$

The deficit in the next downstream reach (KP 100 to KP 80) can be computed with the Streeter-Phelps formula

$$D = 1.894e^{-\frac{1.842}{34.819}x} + \frac{0.514(16.667)}{0.764 - 1.842} \left(e^{-\frac{1.842}{34.819}x} - e^{-\frac{0.764}{34.819}x} \right)$$

For example, at KP 80, the deficit would be

$$D = 1.894e^{-\frac{1.842}{34.819} \cdot 20,000} + \frac{0.514(16.667)}{0.764 - 1.842} \left(e^{-\frac{1.842}{34.819} \cdot 20,000} - e^{-\frac{0.764}{34.819} \cdot 20,000} \right) = 3.02 \text{ mg L}^{-1}$$

which corresponds to an oxygen concentration of

$$o = 8.987 - 3.02 = 5.97 \text{ mg L}^{-1}$$

The deficit in the next downstream reach (KP 80 to KP 60) can be computed as

$$D = 5.97e^{-\frac{1.842}{34.819}(x-20,000)} + \frac{0.514(10.75)}{0.514 - 1.842} \left(e^{-\frac{1.842}{34.819}(x-20,000)} - e^{-\frac{0.514}{34.819}(x-20,000)} \right)$$

At KP 60 (that is, 40 km downstream from the point source), the deficit would be

$$D = 5.97e^{-\frac{1.842}{34.819}(40,000-20,000)} + \frac{0.514(10.75)}{0.514 - 1.842} \left(e^{-\frac{1.842}{34.819}(40,000-20,000)} - e^{-\frac{0.514}{34.819}(40,000-20,000)} \right)$$

$$= 2.70 \text{ mg L}^{-1}$$

which corresponds to an oxygen concentration of

$$o = 8.987 - 2.70 = 6.29 \text{ mg L}^{-1}$$

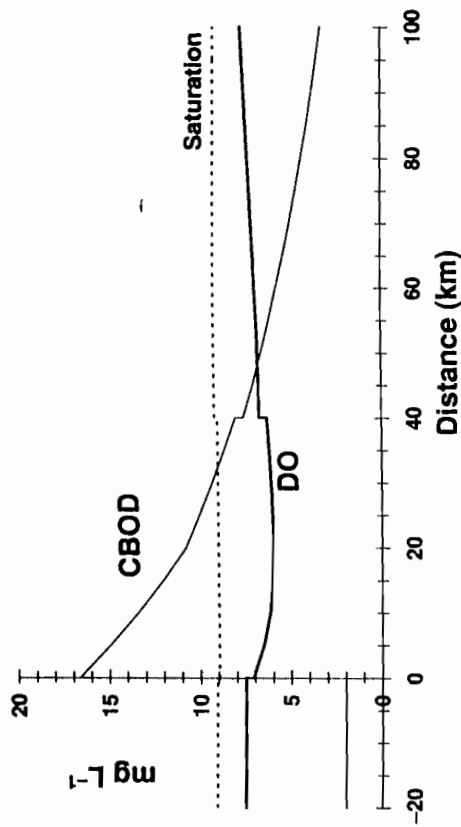


FIGURE E21.2

Now another mass balance is taken to account for the inflowing tributary,

$$L_0 = \frac{540,000(6.29) + 100,000(5)}{640,000} = 2.43 \text{ mg L}^{-1}$$

This value, along with the mass balance for CBOD, serves as the boundary condition for the next reach. The final results are summarized in Fig. E21.2 and the following table:

KP	CBOD (mg L ⁻¹)	Deficit (mg L ⁻¹)	DO (mg L ⁻¹)
-10	2.000	1.592	7.500
0	16.667	1.894	7.093
10	13.384	2.814	6.173
20	10.748	3.022	5.965
30	9.274	2.918	5.997
40	7.532	2.433	6.710
50	6.553	2.391	6.752
60	5.700	2.260	6.883
70	4.959	2.085	7.059
80	4.314	1.891	7.252
90	3.753	1.696	7.447
100	3.265	1.509	7.635

21.5 ANALYSIS OF THE STREETER-PHELPS MODEL

In this section we scrutinize the simple Streeter-Phelps model to try to gain some insight into its behavior. This is extremely important for model calibration. For simplicity the following analysis is written in terms of travel time. Recall that the basic equation is

$$D = D_0 e^{-k_d t} + \frac{k_d L_0}{k_a - k_r} (e^{-k_r t} - e^{-k_d t}) \quad (21.12)$$

The critical travel time can be determined by differentiating Eq. 21.12, setting the result equal to zero, and solving for

$$t_c = \frac{1}{k_a - k_r} \ln \left\{ \frac{k_a}{k_r} \left[1 - \frac{D_0(k_a - k_r)}{k_d L_0} \right] \right\} \quad (21.13)$$

The critical deficit can be determined as

$$D_c = \frac{k_d L_0}{k_a} \left\{ \frac{k_a}{k_r} \left[1 - \frac{D_0(k_a - k_r)}{k_d L_0} \right] \right\}^{-\frac{k_r}{k_a - k_r}} \quad (21.14)$$

These results indicate that the presence of an initial deficit causes the critical deficit to be larger and farther downstream from the discharge. To gain further insight, let us now simplify the analysis by assuming that D_0 is zero. For this case Eqs. 21.13 and 21.14 reduce to

$$t_c = \frac{1}{k_a - k_r} \ln \frac{k_a}{k_r} \quad (21.15)$$

$$\text{and} \quad D_c = \frac{k_d L_0}{k_a} \left(\frac{k_a}{k_r} \right)^{-\frac{k_r}{k_a - k_r}} \quad (21.16)$$

Now some additional general characteristics emerge. First, notice that the critical time depends only on the removal and the reaeration rates. To understand this relationship, note the plot of t_c versus k_r for various values of k_a in Fig. 21.4. This leads to the general conclusion that an increase of either one or both of these parameters means that the critical point on the sag will move closer to the source.

Second, inspection of Eq. 21.16 shows that the magnitude of the critical deficit is linearly related to k_d and L_0 . This makes sense, because each of these parameters contributes linearly to the amount of BOD that is available for decomposition.

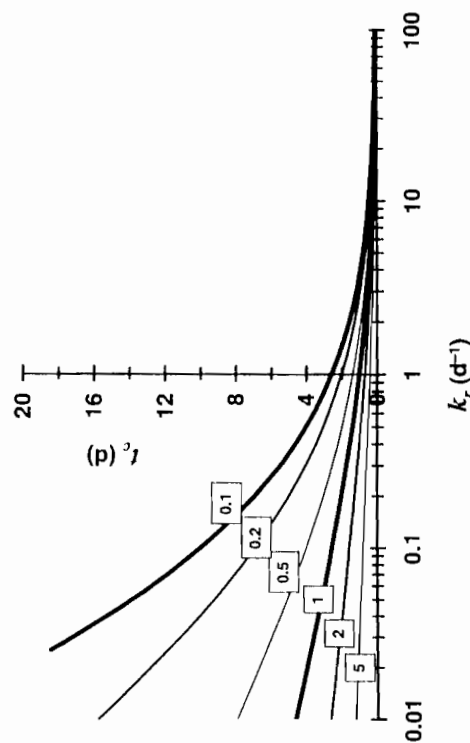


FIGURE 21.4
A plot of t_c versus k_r for various values of k_a (boxed numbers).

21.6 CALIBRATION

The relationships developed in the previous section can be extremely useful in model calibration for the case where the critical time and deficit are known. For example if the initial BOD and reaeration are predetermined, Eqs 21.15 and 21.16 can be used to calculate values for k_d and k_r .

EXAMPLE 21.3. PARAMETER ESTIMATION. A point source and a receiving stream at an altitude of 1524 m (5000 ft) above sea level have the following characteristics:

Values	Point source	River
Flow ($\text{m}^3 \text{s}^{-1}$)	0.7078	3.398
Temperature ($^{\circ}\text{C}$)	22.7	14.6
BOD (mg L^{-1})	40	5
DO (mg L^{-1})	2	8

For 32 km below the discharge point, the stream has a relatively constant width of 61 m and a mean depth of 1.83 m. The following DO measurements are available:

Downstream distance (km)	DO (mg L^{-1})	Downstream distance (km)	DO (mg L^{-1})
1.61	2.9	16.09	4.8
3.22	1.7	19.31	5.6
4.83	1.5	22.53	6.2
6.44	1.7	25.75	6.7
8.05	2.1	28.97	7.0
9.66	2.7	32.19	7.3
12.87	3.8		

Determine the total removal rate (k_t) and the deoxygenation rate (k_d) from the data, assuming that the O'Connor-Dobbins formula adequately predicts reaeration.

Solution: Mass balances at the outfall can be used to determine: $L_0 = 11 \text{ mg L}^{-1}$, $o_0 = 6.97 \text{ mg L}^{-1}$, and $T_0 = 16^{\circ}\text{C}$. The temperature can be used in conjunction with Eqs. 19.32 and 19.39 to compute $\alpha_{16} = 8.143 \text{ mg L}^{-1}$. This value, in turn, can be employed to calculate $D_0 = 1.173 \text{ mg L}^{-1}$. Finally the O'Connor-Dobbins formula along with a temperature correction can be used to calculate $k_r = 0.2775 \text{ d}^{-1}$.

The DO data can be used to determine deficit downstream from the discharge. From this information it can be estimated that the critical deficit of 6.643 mg L^{-1} occurs at a travel time of 1.517 d. Substituting values into Eq. 21.13 gives

$$1.517 = \frac{1}{0.2775 - k_r} \ln \left\{ \frac{0.2775}{k_r} \left[1 - \frac{1.173(0.2775 - k_r)}{k_d(11)} \right] \right\}$$

and into Eq. 21.14,

$$6.643 = \frac{k_d(11)}{0.2775} \left\{ \frac{0.2775}{k_r} \left[1 - \frac{1.173(0.2775 - k_r)}{k_d(11)} \right] \right\}^{-0.2775 - k_r}$$

These two nonlinear equations can be solved simultaneously for k_r and $k_d = 1.159$ and 0.97 d^{-1} . These values, along with the other model parameters, can then be used in conjunction with the Streeter-Phelps model to compute the fit shown in Fig. E21.3.

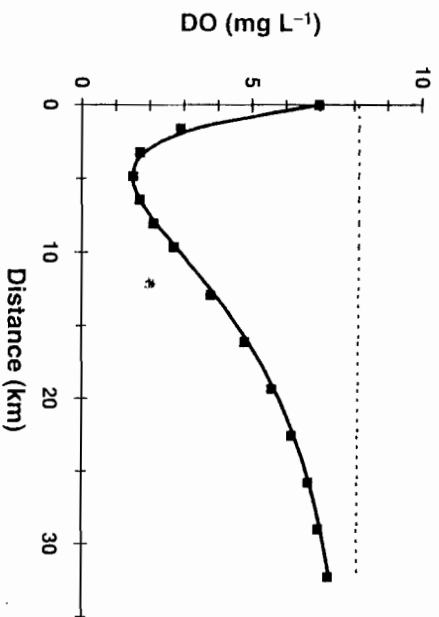


FIGURE E21.3

21.7 ANAEROBIC CONDITION

It is possible that the magnitude of a BOD discharge is so great that a stream will be devoid of oxygen. For such cases the Streeter-Phelps model must be modified. Gundelach and Castillo (1970) have developed a nice analysis for such cases.

For simplicity we again express the equations in terms of travel time. In addition we assume that $k_d = k_r$ (that is, there are no settling losses). For this situation the Streeter-Phelps model is

$$L = L_0 e^{-k_d t} \quad (21.17)$$

$$D = D_0 e^{-k_d t} + \frac{k_d L_0}{k_d - k_d} \left(e^{-k_d t} - e^{k_d t} \right) \quad (21.18)$$

The point where the system goes anaerobic can be determined by solving Eq. 21.18 for t with $D = 0$. This must be done numerically. In other words, the solution amounts to finding the smallest root of

$$f(t) = D_0 e^{-k_d t} + \frac{k_d L_0}{k_d - k_d} \left(e^{-k_d t} - e^{k_d t} \right) - 0_s \quad (21.19)$$

When this point is reached, oxygen depletion can no longer proceed at the rate of $k_d L$. Rather it will be limited by the rate at which oxygen passes across the air-water interface via reaeration. That is,

$$\frac{dL}{dt} = -k_a O_2 \quad (21.20)$$

Thus once the oxygen is totally depleted, the reaction becomes zero-order. Using the initial condition that $L = L_i$ at $t = t_i$, we can calculate the BOD in the stretch from t_i to t_f as

$$L = L_i - k_a O_2 (t - t_i) \quad (21.21)$$

Consequently as with all zero-order reactions, the BOD is reduced linearly.

Finally we must determine where the anaerobic zone ends. At this point,

$$k_a O_2 = k_d L_f \quad (21.22)$$

Combining Eqs. 21.21 and 21.22 yields

$$t_f = t_i + \frac{1}{k_d} \frac{k_d L_i - k_a O_2}{k_a O_2} \quad (21.23)$$

EXAMPLE 21.4. ANAEROBIC CONDITIONS. Repeat Example 21.3, but double the BOD concentration of the treatment plant. That is, increase the point source of BOD to 80 mg L^{-1} . Also assume that no settling losses occur (that is, $k_r = k_d = 0.97 \text{ d}^{-1}$). All other values are assumed to be the same as for Example 21.3.

Solution: The initial CBOD concentration must be recalculated to reflect the higher concentration of the wastewater. The result is 17.93 mg L^{-1} . This value, along with the other parameters, can be substituted into Eq. 21.18 to give

$$D = 1.173 e^{-\frac{0.2775}{3182}x} + \frac{0.97(17.93)}{0.2775 - 0.97} \left(e^{-\frac{0.97}{3182}x} - e^{-\frac{0.2775}{3182}x} \right)$$

When we set $D = 0$, this equation can be used to estimate that the oxygen concentration will reach zero at a travel time of 0.589 d , which corresponds to a distance of $x = 1.87 \text{ km}$. At this point the BOD will have decomposed to a level of 10.13 mg L^{-1} . This serves as the value of L_i to be used in conjunction with Eqs. 21.21 and 21.23.

The length of the anoxic stretch can be determined via Eq. 21.23:

$$t_f = 0.589 + \frac{1}{0.97} \frac{0.97(10.13) - 0.2775(8.143)}{0.2775(8.143)} = 4.04 \text{ d}$$

which corresponds to $x = 12.85 \text{ km}$. The BOD at this point can be calculated with Eq. 21.21.

$$L = 10.13 - 0.2775(8.143)(4.04 - 0.589) = 2.33 \text{ mg L}^{-1}$$

This then provides a boundary condition to calculate the remainder of the concentrations using the normal Streeter-Phelps model. The final results for both BOD and oxygen are shown in Fig. 21.5. Notice how for BOD, after an initial steep exponential drop, the degradation slows to a linear decrease in the anoxic region. After oxygen moves above zero, the more rapid exponential decrease returns.

For DO, along with the simulation corrected for anoxia, we also display the original result for the low-BOD load as well as a high-BOD-load simulation where we make no correction for anoxia (that is, we allow the model to go negative). Note how the correction causes the zone of low oxygen to expand because of the depressed decomposition that occurs.

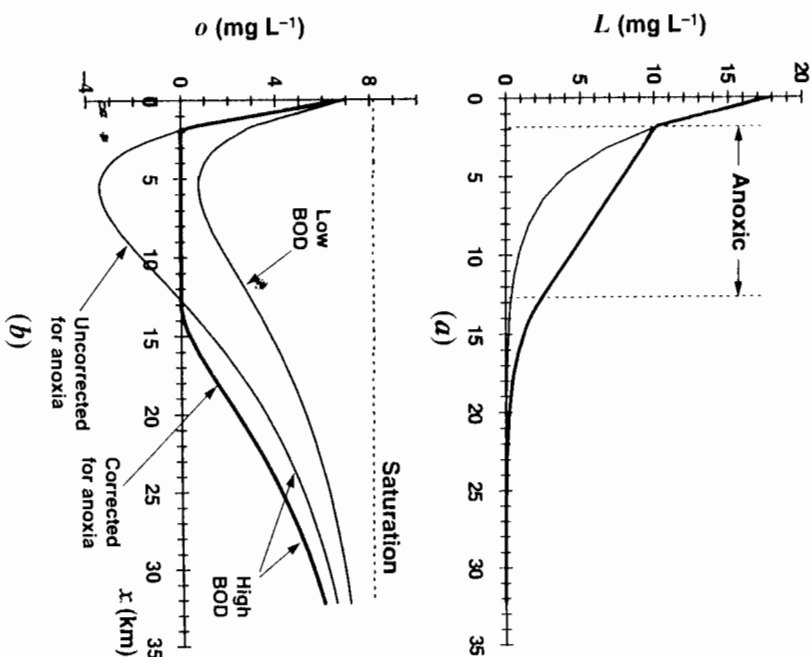


FIGURE 21.5 Plots of (a) BOD and (b) dissolved oxygen down-stream from a point source where oxygen levels drop to zero.

21.8 ESTUARY STREETER-PHELPS

For a dispersive system such as an estuary, the Streeter-Phelps equation can be written as

$$0 = E \frac{d^2 L}{dx^2} - U \frac{dL}{dx} - k_r L \quad (21.24)$$

$$\text{and} \quad 0 = E \frac{d^2 D}{dx^2} - U \frac{dD}{dx} + k_d L - k_a D \quad (21.25)$$

The solution for BOD is

$$L = L_0 e^{j_1 r x} \quad x \leq 0 \quad (21.26)$$

$$L = L_0 e^{j_2 r x} \quad x \geq 0 \quad (21.27)$$

and for oxygen deficit is

$$D = \frac{k_d}{k_a - k_r} \frac{W}{Q} \left(\frac{e^{j_1 r x}}{\alpha_r} - \frac{e^{j_2 r x}}{\alpha_a} \right) \quad x \leq 0 \quad (21.28)$$

$$D = \frac{k_d}{k_a - k_r} \frac{W}{Q} \left(\frac{e^{j_2 x}}{\alpha_r} - \frac{e^{j_1 x}}{\alpha_a} \right) \quad x \geq 0 \quad (21.29)$$

where

$$L_0 = \frac{W}{\alpha_r Q} \quad (21.30)$$

$$\alpha_r = \sqrt{1 + \frac{4k_r E}{U^2}} \quad (21.31)$$

$$\alpha_a = \sqrt{1 + \frac{4k_a E}{U^2}} \quad (21.32)$$

$$j_{1r} = \frac{U}{2E} (1 \pm \alpha_r) \quad (21.33)$$

$$j_{1a} = \frac{U}{2E} (1 \pm \alpha_a) \quad (21.34)$$

The solution is displayed in Fig. 21.6, along with the plug-flow case. Notice how dispersion spreads out the sag.

In a fashion similar to the derivation of Eqs. 21.15 and 21.16, critical distances and deficits can be determined, as in

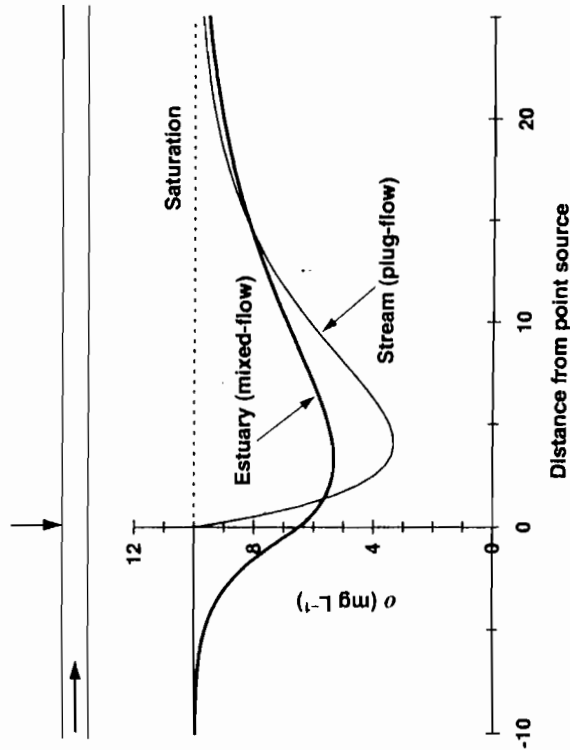


FIGURE 21.6
Plot of dissolved oxygen for a point source into an estuary. The solution for the plug-flow case is shown for comparison.

$$x_c = \frac{\ln \left(\frac{\alpha_r j_{2a}}{\alpha_a j_{2r}} \right)}{j_{2r} - j_{2a}} \quad (21.35)$$

$$\text{and} \quad D_c = \frac{k_d}{k_a - k_r} \frac{W}{Q} \left(\frac{e^{j_{2a} x_c}}{\alpha_r} - \frac{e^{j_{2r} x_c}}{\alpha_a} \right) \quad (21.36)$$

Observe that, as with the plug-flow case, the critical deficit is linearly related to the loading and the deoxygenation rate. In addition the location is solely dependent on the removal and reaeration rates. As with the plug-flow model, higher removal and reaeration rates tend to move the critical deficit closer to the outfall.

PROBLEMS

21.1. A tanker truck careens off the road and dumps 30,000 L of glucose syrup into a small mountain lake. The concentration of the syrup is 100 g-glucose L^{-1} . The lake has the following characteristics in the period immediately following the spill: residence time = 30 d, depth = 5 m, area = $5 \times 10^4 \text{ m}^2$, altitude = 11,000 ft, wind speed = 2.235 $m \text{ s}^{-1}$, and temperature = 10°C . Note that the lake is assumed to be completely mixed, and has zero BOD and is at saturation prior to the spill. Compute the oxygen level in the lake during the period following the spill.

21.2. Some organic matter is added to a closed reactor so that its initial concentration is 4 $\text{mg } L^{-1}$ of TOC. As the organic matter decays, you know (from a previous study) that it will deoxygenate the water at a rate of 0.2 d^{-1} at a temperature of 20°C .

(a) If the present experiment is conducted at 15°C , predict the concentration of BOD versus time in the reactor. Plot your results.

(b) Determine the dissolved oxygen and the deficit versus time within the closed reactor. Plot your results.

(c) Open the reactor to the atmosphere and repeat parts (a) and (b). Note that your laboratory is located at an altitude of 2 km and there is a gentle breeze of 1 mps blowing over the reactor during the experiment.

21.3. You make the following measurements below a sewage plant outfall into a stream at sea level:

Travel time (d)	0	2	4	6	8	10	12	16	20
CBOD ($\text{mg } L^{-1}$)	50	38.9	30.0	23.0	17.7	13.9	11.4	6.7	3.7
DO ($\text{mg } L^{-1}$)	10.0	5.3	2.7	1.6	1.3	1.6	2.2	4.0	5.7

If the water temperature is 10°C , determine the reaeration and deoxygenation rates.

21.4. A fresh stream having the characteristics $Q_f = 7 \text{ cms}$, $S_f = 1 \text{ ppt}$, $o_f = 12 \text{ mg } L^{-1}$, and $T_f = 10^\circ\text{C}$ discharges into a saline stream ($Q_s = 2 \text{ cms}$, $S_s = 15 \text{ ppt}$, $o_s = 3 \text{ mg } L^{-1}$, and $T_s = 18^\circ\text{C}$). Determine the oxygen and the oxygen deficit assuming complete mixing at the confluence of the two streams. Note that the elevation is 1.5 km.

21.5. A waste source ($Q_w = 1$ cms, $L_w = 25$ mg L⁻¹, $o_w = 2$ mg L⁻¹, $T_w = 25^\circ\text{C}$) discharges into a stream ($Q_r = 10$ cms, $L_r = 2$ mg L⁻¹, $o_r = 10$ mg L⁻¹, and $T_r = 15^\circ\text{C}$). Downstream, the velocity is 0.3 mps and the depth is 0.3 m. Calculate the profiles of both BOD and oxygen downstream, assuming that the stream is located at 5500 ft elevation and that the BOD decays at a rate of 1 d⁻¹. Determine the value and the location of the maximum deficit.

21.6. Determine the profiles of BOD and dissolved oxygen for the following sea-level stream:

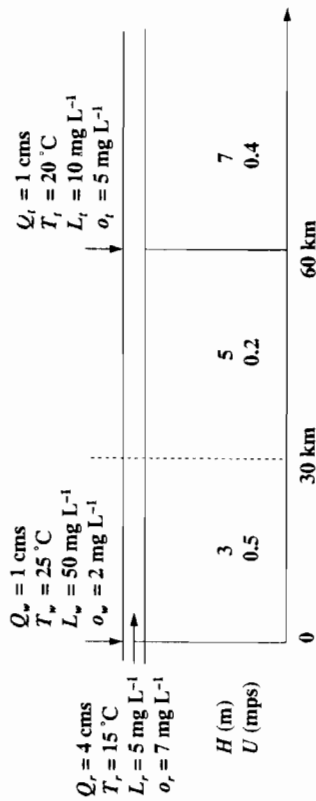


FIGURE P21.6

LECTURE OVERVIEW: I focus on the effect of distributed sources of BOD and deficit on the Streeter-Phelps model. The initial sections describe cases in which the distributed sources contribute pollutant mass but do not add significant quantities of flow. I follow this with a description of models of flow contributing sources.

To this juncture I have focused on point sources. Now let's look at nonpoint or diffuse sources. As the name implies, these inputs enter the system in a diffuse manner. For one-dimensional streams and estuaries this means that the loadings enter the system along its length.

In this lecture I focus on two types of diffuse sources: those that do not contribute flow and those that do. Before doing that I'll briefly describe how the rates of diffuse source contributions are parameterized.

22.1 PARAMETERIZATION OF DISTRIBUTED SOURCES

Recall that in Lec. 9, I showed how a steady-state mass balance for a distributed source (Fig. 22.1a) could be written for a plug-flow system with uniform hydrology and geometry.

$$0 = -\frac{dc}{dt} - kc + S_d \quad (22.1)$$

where S_d = rate of the distributed source (g m⁻³ d⁻¹) and t = travel time for a plug-flow system. If $c = 0$ at $t = 0$, then