LECTURE 28

The Eutrophication Problem and Nutrients

LECTURE OVERVIEW: I provide background information on the eutrophication problem. After an overview of the symptoms of eutrophication, I discuss the primary nutrients that stimulate the process. I also describe plant stoichiometry and the nitrogen:phosphorus ratio.

Several times a year I fertilize my lawn and garden so I can grow green grass and lots of fresh vegetables. In a similar way the addition of nutrients to a natural water stimulates plant growth. In small quantities this can be a good thing. For example because of the higher human population of its drainage basin, Lake Michigan receives more nutrients than Lake Superior. Consequently Lake Michigan has more plant growth, which ultimately allows it to support more game fish.

However, as with most everything in life, there can always be too much of a good thing. When lakes, streams, and estuaries are overfertilized, the resulting excessive plant growth can become a serious water-quality problem.

This "overfertilization" phenomenon is generally referred to as ***eutrophication***. This terminology was originally coined to describe the natural aging process whereby a lake is transformed from a lake to a marsh to a meadow. This process can take thousands of years to occur naturally. However, the process can be greatly quickened by excess nutrients from human activities. This accelerated process is sometimes called ***cultural eutrophication***.

Water bodies are often classified as to their trophic state. The general terms are

- Oligotrophic (poorly nourished)
- Mesotrophic (moderately nourished)
- Eutrophic (well-nourished)
- Hypereutrophic (overnourished)

Although such terms were originally developed for and are most commonly applied to lakes, they are also appropriate descriptors of streams and estuaries.

28.1 THE EUTROPHICATION PROBLEM

In general, eutrophication can have a number of deleterious effects on water bodies. These include:

- *Quantity.* The profuse growth of floating plants decreases water clarity, and some species form unsightly scums. Further, certain floating plants can clog filters at water treatment plants, and overgrowth of rooted plants can hinder navigation and recreation by clogging waterways.
- *Chemistry.* Plant growth and respiration can affect the system's water chemistry. Most notably, oxygen and carbon dioxide levels are directly impacted by plant activity. Oxygen has implications related to the survival of organisms such as fish. In particular the bottom waters of thermally stratified systems can become totally devoid of oxygen due to the decomposition of dead plants. Carbon dioxide can impact pH.
- *Biology.* Eutrophication can alter the species composition of an ecosystem. Native biota may be displaced as the environment becomes more productive. Certain species of algae cause taste and odor problems in drinking water. Further, certain blue-green algae can be toxic when consumed by animals. Many of these problems become prominent as the water body becomes more eutrophic.

Now that we have a feeling for the types of problems caused by eutrophication, we must figure out how it works. The first step in this process involves the inorganic nutrients that serve as the raw materials from which plant biomass is synthesized.

28.2 NUTRIENTS

Inorganic nutrients provide chemical building blocks for life in aquatic systems. Some are required in large quantities for cell development and hence are called ***macronutrients***. These are carbon, oxygen, nitrogen, phosphorus, sulfur, silica, and iron. Smaller quantities of ***micronutrients***, such as manganese, copper, and zinc, are also necessary. Water-quality modeling has focused on four macronutrients: phosphorus, nitrogen, carbon, and silica.

28.2.1 Phosphorus

Phosphorus is essential to all life. Among other functions, it has a critical role in genetic systems and in the storage and transfer of cell energy.

From a water-quality perspective, phosphorus is important because it is usually in short supply relative to the other macronutrients. This scarcity is due to three primary factors:

1. It is not abundant in the earth's crust. Further, the phosphate minerals that do exist are not very soluble.
2. It does not exist in a gaseous form. Thus, in contrast to carbon and nitrogen, there is no gaseous atmospheric source.
3. Finally phosphate tends to sorb strongly to fine-grained particles. The settling of these particles, along with sedimentation of organic particles containing phosphorus, serves to remove phosphorus from the water to the bottom sediments. For cases where the water in contact with the sediments contains oxygen, such sediment phosphorus becomes chemically trapped.

Although phosphorus is naturally scarce, many human activities result in phosphorus discharge to natural waters. Human and animal wastes both contain substantial amounts of phosphorus. In the recent past the former has been supplemented by detergent phosphorus. In addition nonpoint sources from agricultural and urban land both contribute excess phosphorus. Part of the enhancement of diffuse sources is due to fertilizers and other phosphorus-containing chemicals associated with human land use. Moreover, human uses lead to soil erosion, which also enhances phosphorus transport into waters.

Phosphorus in natural waters can be subdivided in several ways. One scheme, which stems from conventional measurement techniques and modeling necessity, is (Fig. 28.1)

- *Soluble reactive phosphorus (SRP).* Also called orthophosphate or soluble inorganic P, this is the form that is readily available to plants. It consists of the species $H_2PO_4^-$, HPO_4^{2-}, and PO_4^{3-}.
- *Particulate organic P.* This form mainly consists of living plants, animals, and bacteria as well as organic detritus.
- *Nonparticulate organic P.* These are dissolved or colloidal organic compounds containing phosphorus. Their primary origin is the decomposition of particulate organic P.
- *Particulate inorganic P.* This category consists of phosphate minerals (e.g., apatite phosphorus), sorbed orthophosphate (e.g., on clays), and phosphate complexed with solid matter (e.g., calcium carbonate precipitates or iron hydroxides).
- *Nonparticulate inorganic.* This group includes condensed phosphates such as those found in detergents.

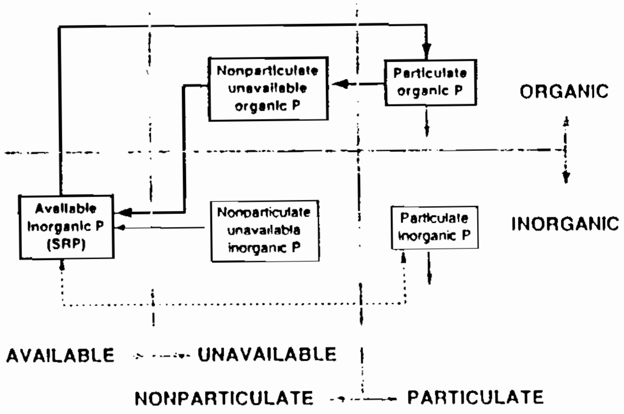

FIGURE 28.1
Forms of phosphorus found in natural waters. The principal forms involved in the production/decomposition life cycle are shown in bold.

Phosphorus is measured in several ways. Soluble reactive phosphorus (SRP) is measured by adding ammonium molybdate, which forms a colored complex with the phosphate. Organic phosphorus is not detected by the test unless it is first hydrolyzed, that is, converted to orthophosphate by digestion with heat and a strong acid. In addition filtering can be used to divide between particulate and nonparticulate components. Finally application of the SRP method to a digested, unfiltered sample provides a means to measure all the phosphorus in a sample. As we will see later in this lecture and the next, such a ***total phosphorus (TP)*** measurement has been used widely to quantify eutrophication.

As mentioned above, the partitioning in Fig. 28.1 is based on available measurement techniques and modeling necessity. The distinction between particulate and nonparticulate forms is made so that the former can selectively be removed by settling. The division of available phosphorus from the other species is made because it is the only form that is directly available for plant growth. It should be understood that the other forms are not absolutely "unavailable." Rather, they must first be converted to SRP before they can be consumed by plants.

It also should be noted that Fig. 28.1 is not the last word regarding phosphorus segmentation. In fact from a strictly scientific basis, each of the compartments could be broken down into finer detail. Conversely from the perspective of water-quality modeling, some of the compartments are often consolidated whereas others are broken down into finer segments. For example the distinction between inorganic and organic unavailable forms is not usually made. That is, they are lumped into unavailable particulate P and unavailable nonparticulate groupings. An example of more refinement is that living particulate organic P is often distinguished from nonliving forms. In other words, groups of organisms (such as phytoplankton, zooplankton, etc.) are modeled separately. In such cases the amount of phosphorus contained in these groups would have to be subtracted from the particulate P compartment. We return to this topic when we build models in a later lecture.

28.2.2 Nitrogen

I have already described the nitrogen cycle in Lec. 23. Recall that the primary forms are

- Free nitrogen (N_2)
- Ammonium (NH_4^+)/ammonia (NH_3)
- Nitrite (NO_2^-)/nitrate (NO_3^-)
- Organic nitrogen

As depicted in Fig. 28.2, the organic nitrogen can be broken down further into particulate and dissolved components.

Some of the major processes governing the dynamics of these groups are

- *Ammonia and nitrate assimilation.* This includes the uptake of inorganic nitrogen by phytoplankton. Although phytoplankton utilize both ammonia and nitrate, their preference for the former has been demonstrated (Harvey 1955, Walsh and Dugdale 1972, Bates 1976).

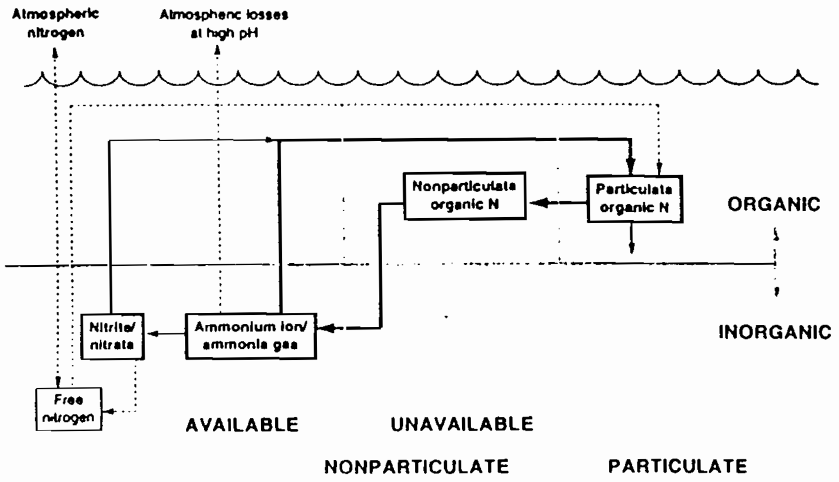

FIGURE 28.2
Forms of nitrogen found in natural waters. The principal forms involved in the production/decomposition life cycle are shown in bold.

- *Ammonification.* This is the transformation of organic nitrogen to ammonia. This is a complicated process involving several mechanisms, including bacterial decomposition, zooplankton excretion, and direct autolysis after cell death.
- *Nitrification.* This is the oxidation of ammonia to nitrite and nitrite to nitrate via the action of a select group of aerobic bacteria. This process utilizes oxygen and typically is represented by first-order reactions. The fact that the transformation of nitrite to nitrate is relatively fast supports the use of a single functional group to encompass both nitrate and nitrite in some nutrient/food-chain models. For these cases the nitrification process would be represented kinetically by the oxidation of ammonia.
- *Denitrification.* Under anaerobic conditions—for example in the sediments and the anoxic hypolimnia of some lakes—nitrate can serve as an electron acceptor for certain bacteria. Nitrite is formed as an intermediate, with the principal end product being free nitrogen.
- *Nitrogen fixation.* A number of organisms can fix elemental nitrogen. An important group from the standpoint of nutrient/food-chain modeling is blue-green algae possessing heterocysts. These organisms are important because, for lakes with high phosphorus loadings, phytoplankton growth can depress nitrogen levels to the point where nonfixing algae will become nitrogen-limited. The ability of the blue-green algae to utilize free nitrogen gives them a competitive advantage in such situations. The resulting dominance by the blue-greens has implications to water quality since many species have objectionable characteristics; for example they form floating scums.

Although nitrogen is just as necessary for life as phosphorus, they differ in three ways:

- *Nitrogen has a gas phase.* Further, as mentioned above, certain blue-green algae are capable of fixing free nitrogen. This gives them a competitive advantage in situations where other forms of nitrogen are in short supply. This state of affairs can sometimes occur when advanced treatment includes nitrogen removal. In such cases blue-green algae can become dominant.
- *Inorganic forms of nitrogen do not sorb as strongly to particulate matter as does phosphorus.* Consequently, although particulate forms of nitrogen are carried to the sediments by settling, they are more easily introduced back into the water. In addition inorganic forms of nitrogen (particularly nitrate) are more mobile in groundwater.
- *Denitrification represents a purging mechanism that does not occur for phosphorus.* Because it occurs only in the absence of oxygen, denitrification is insignificant for many surface waters. However, for productive systems where denitrification can occur in anoxic sediments, a deficiency of nitrogen can be created.

As with phosphorus, nitrogen discharges to natural water result from human activities. Human and animal wastes both contain substantial amounts of nitrogen. In addition nonpoint sources from agricultural and urban land both contribute excess nitrogen. As mentioned above, because forms such as nitrate do not associate strongly with solid matter, they can be easily transmitted to surface waters along with groundwater flow.

Most of the aforementioned characteristics mean that phosphorus has usually been identified as the primary controllable nutrient governing the eutrophication process in fresh waters. However, productive estuaries can tend to be nitrogen limited.

28.2.3 Carbon

Carbon can play three roles in water-quality modeling:

- *Nutrient.* In the same way as phosphorus and nitrogen, carbon can be thought of as a nutrient. However, in most cases modelers usually assume that carbon cannot limit algal growth, although some models (Chen 1970, Chen and Orlob 1975) do allow for potential limitation. Studies dealing with carbon-limited algal cultures raise questions regarding the form of carbon needed for photosynthesis by different algae (Goldman et al. 1974, King and Novak 1974). It also has been suggested that the relative abilities of green and blue-green algae to use various forms of inorganic carbon could partially explain the succession from greens to blue-greens in enriched natural waters (King 1972, Shapiro 1973). These and other developments (see references in Goldman et al. 1974) indicate that control of primary production by carbon limitation could be important in certain systems.
- *Biomass.* Because it usually constitutes a large component of organic compounds, carbon is often used as a measure of biomass.
- *Pollutant.* Finally carbon is an important factor in other pollution problems beyond eutrophication. First, as already discussed in great detail in Part IV, the decomposition of organic carbon can greatly affect a system's oxygen concentration. Second, it is known that many toxicants preferentially associate with organic matter. Thus the dynamics of toxics in the environment is intimately connected with the generation, transport, and fate of organic carbon. Finally naturally produced organic carbon can, itself, be transformed into a toxic compound. For example chlorine can react with organic compounds to form toxic trihalomethanes.

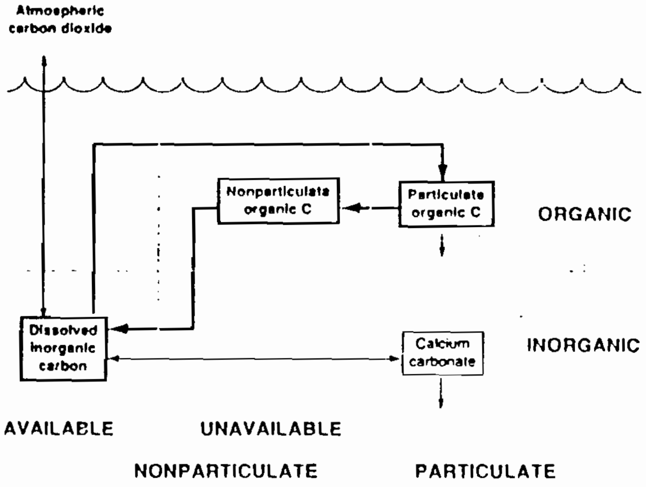

FIGURE 28.3
Forms of carbon found in natural waters. The principal forms involved in the production/decomposition life cycle are shown in bold.

Figure 28.3 is an effort to represent the carbon cycle in natural waters. Note that the dissolved inorganic carbon compartment actually consists of several species: carbon dioxide (CO_2), bicarbonate (HCO_3^-), and carbonate (CO_3^-).

28.2.4 Silicon

Although it might be considered a minor nutrient, silicon has significance in the dynamics of phytoplankton because of its importance as a major structural element in the cells of an important phytoplankton group—the diatoms. These organisms use dissolved reactive silicon [mainly as $Si(OH)_4$] to build a *frustule* or "glass wall" that surrounds the cell. Silicon in frustules is not available to other diatoms, and ambient concentrations of available silicon can therefore become low enough to limit further growth of these algae.

In most modeling work to date, silicon has not been simulated. Where it has been included, silicon has been treated as one compartment, such as dissolved inorganic silicon (Lehman et al. 1975), or two compartments, such as available silicon and unavailable silicon (Scavia 1980).

28.3 PLANT STOICHIOMETRY

Aside from nutrients, the other key part of the eutrophication process is the food chain. As depicted previously in Fig. 19.1, the exchange between the two components represents a cycle. That is, production converts inorganic nutrients into organic matter whereas decomposition reverses the process.

An important factor in the process is the stoichiometric composition of organic matter. Although the composition varies, the dry-weight[†] composition can be idealized as in the following detailed representation of the photosynthesis/respiration process (Stumm and Morgan 1981):[‡]

$$106CO_2 + 16NH_4^+ + HPO_4^{2-} + 108H_2O \rightleftharpoons \underset{\text{"Algae"}}{C_{106}H_{263}O_{110}N_{16}P_1} + 107O_2 + 14H^+ \qquad (28.1)$$

This formula can be used to determine the mass ratios of carbon to nitrogen to phosphorus:

$$\begin{array}{ccccc} C & : & N & : & P \\ 106 \times 12 & : & 16 \times 14 & : & 1 \times 31 \\ 1272 & : & 224 & : & 31 \end{array} \qquad (28.2)$$

It is also known that plant protoplasm is about 1% phosphorus on a dry-weight basis. Therefore we can normalize the ratios to the mass of phosphorus and express the results as percentages of dry weight,

$$\begin{array}{ccccc} C & : & N & : & P \\ 40\% & : & 7.2\% & : & 1\% \end{array} \qquad (28.3)$$

Thus a gram dry weight of organic matter contains approximately 10 mg of phosphorus, 72 mg of nitrogen, and 400 mg of carbon. Finally it should be noted that dry-weight biomass has a density of about 1.27 g cm^{-3} and wet-weight biomass is about 90% water. Figure 28.4 summarizes the fundamental information regarding cell stoichiometry. Additional information, particularly regarding the variability of these numbers, can be found in references such as Bowie et al. (1985).

Although the foregoing information provides a way to break down biomass into its individual components, additional information is required because phytoplankton are often measured in units other than dry weight. In cases where organic carbon is used, the foregoing stoichiometric information can be employed directly. More commonly the phytoplankton are measured as chlorophyll *a*. In general the chlorophyll-to-carbon ratio ranges from 10 to 50 μgChl mgC^{-1}. The lower value is usually typical of well-illuminated waters such as oligotrophic systems. For such systems less chlorophyll is required because of the high solar radiation. In contrast, waters with less light, such as eutrophic and turbid systems, would tend to have phytoplankton with a higher chlorophyll content.

Dry weight means the weight of the organic matter after it has been dehydrated.

This formula holds when ammonium is the source of inorganic nitrogen. For the case where nitrate is the source, the reaction is modified.

$$106CO_2 + 16NO_3^- + HPO_4^{2-} + 122H_2O + 18H^+ \rightleftharpoons C_{106}H_{263}O_{110}N_{16}P_1 + 138O_2$$

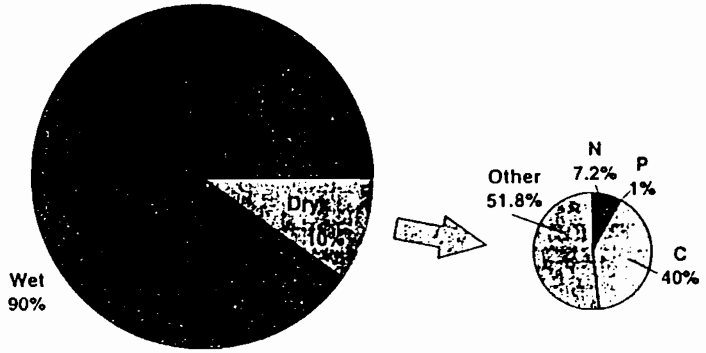

FIGURE 28.4
Pie diagram showing the percentages of nutrients and water constituting the average phytoplankton biomass.

EXAMPLE 28.1. PHYTOPLANKTON STOICHIOMETRY. Suppose that a lake has a volume of 1 × 10^6 m^3 and a phytoplankton concentration of 10 μg L^{-1} of chlorophyll *a*. If the carbon-to-chlorophyll ratio is 25 μgChl mgC^{-1} and all the other stoichiometry follows Fig. 28.4: (*a*) Reexpress the phytoplankton concentration as organic carbon. (*b*) If the phytoplankton are decomposing at a rate of 0.1 d^{-1}, what is the resulting rate of oxygen demand in g m^{-3} d^{-1}? (*c*) What is the rate of release of nitrogen and phosphorus in g d^{-1}?

Solution:

(*a*) The phytoplankton concentration as organic carbon can be calculated as

$$10\frac{\text{mgChl}a}{\text{m}^3}\left(\frac{\text{gC}}{25\ \text{mgChl}a}\right) = 0.40\ \text{gC m}^{-3}$$

(*b*) The decomposition of a gram of organic carbon utilizes 2.67 g of oxygen (recall Eq. 19.18). Thus the amount of oxygen consumed can be calculated as

$$r_{oc}k_d c = 2.67\frac{\text{gO}}{\text{gC}}\left(\frac{0.1}{\text{d}}\right)0.40\frac{\text{gC}}{\text{m}^3} = 0.1068\ \text{gO m}^{-3}\ \text{d}^{-1}$$

(*c*) The phosphorus generation rate can be calculated by

$$a_{pa}k_d Va = 1\frac{\text{mgP}}{\text{mgChl}a}\left(\frac{0.1}{\text{d}}\right)1 \times 10^6\ \text{m}^3\left(\frac{\text{mgChl}a}{10\ \text{m}^3}\right)\left(\frac{1\ \text{gP}}{1000\ \text{mgP}}\right) = 1000\ \text{gP d}^{-1}$$

The nitrogen-to-chlorophyll ratio can be computed as

$$a_{na} = 1\frac{\text{mgP}}{\text{mgChl}a}\left(7.2\frac{\text{mgN}}{\text{mgP}}\right) \doteq 7.2\frac{\text{mgN}}{\text{mgChl}a}$$

which can then be employed to determine the nitrogen generation rate,

$$a_{na}k_d Va = 7.2\frac{\text{mgN}}{\text{mgChl}a}\left(\frac{0.1}{\text{d}}\right)1 \times 10^6\ \text{m}^3\left(10\frac{\text{mgChl}a}{\text{m}^3}\right)\left(\frac{1\ \text{gN}}{1000\ \text{mgN}}\right) = 7200\ \text{gN d}^{-1}$$

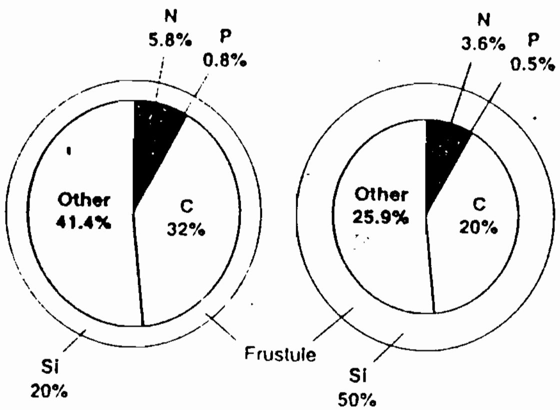

FIGURE 28.5
Pie diagram showing the percentages of nutrients on a dry-weight basis for diatoms with different silicon content in their "glass" cell walls or frustules.

Diatoms differ from other forms of phytoplankton in that silicon makes up a large fraction of their biomass. The percentage of dry weight ranges from 20 to 50% depending on the structure of their frustule. It is usually assumed that the remainder of their dry-weight biomass follows the proportions in Eq. 28.4. Figure 28.5 shows how the dry-weight percentages would decrease as more of the total dry weight is dedicated to silica.

28.4 NITROGEN AND PHOSPHORUS

Because they are the primary controllable nutrients, nitrogen and phosphorus have been the focus of most efforts to control eutrophication.

28.4.1 Nitrogen and Phosphorus Sources

Point sources of nitrogen and phosphorus are summarized in Table 28.1. In general the use of phosphate detergents has had a great impact on the amount of phosphorus in wastewater.

Some typical nonpoint sources are summarized in Table 28.2. Notice that urban and agricultural use greatly increase the export of both nitrogen and phosphorus from the land.

28.4.2 N:P Ratio

We now know that as plants grow they take up inorganic nutrients from the water in proportion to their stoichiometry. In eutrophication management it is often important to identify which of the several nutrients used for plant nutrition ultimately controls the level of plants in the water body. A first cut at identifying this "limiting nutrient" is to compare the levels of the nutrients in the water with the cell stoichiometry.

TABLE 28.1
Amounts of nitrogen and phosphorus in untreated domestic sewage in the United States. Numbers in parentheses represent ranges (from Metcalf and Eddy 1991, Thomann and Mueller 1987)

Nutrient	**Concentration ($mg\ L^{-1}$)**	**Per-capita loading rate ($g\ capita^{-1}\ d^{-1}$)[a]**
Nitrogen	40 (20–85)	23
Organic N	15 (8–35)	8.5
Free ammonia	25 (12–50)	14.2
Phosphorus (with detergents)	8 (4–15)	4.5
Organic P	3 (1–5)	1.7
Inorganic P	5 (3–10)	2.8
Phosphorus (without detergents)	4	2.3

[a] Based on a per-capita flow rate of 0.57 m^3 $capita^{-1}\ d^{-1}$ (150 gal $capita^{-1}\ d^{-1}$).

TABLE 28.2
Nitrogen and phosphorus export rates ($kg\ ha^{-1}\ yr^{-1}$) generated from various nonpoint sources in the United States (numbers in parentheses represent ranges)

Nutrient	**Forest**	**Agricultural**	**Urban**	**Atmospheric**
Nitrogen	3 (1.3–10.2)	5 (0.5–50)	5 (1–20)	24
Phosphorus	0.4 (0.01–0.9)	0.5 (0.1–5)	1 (0.1–10)	1 (0.05–5)

This is most commonly done for nitrogen and phosphorus. A rough rule of thumb for assessing which nutrient is limiting relates to the nitrogen-to-phosphorus ratio. Recall that the ratio of nitrogen to phosphorus in biomass is approximately 7.2. Hence an N:P ratio in the water that is less than 7.2 suggests that nitrogen is limiting. Conversely, higher levels imply that phosphorus will limit plant growth. The rationale behind the ratio is derived in the following example.

EXAMPLE 28.2. N:P RATIOS. Suppose that a batch reactor has an initial concentration of algae of 1 μgChl L^{-1}. If the plants are growing according to first-order kinetics ($k_g = 1\ d^{-1}$), determine how both plant and nutrient concentrations evolve for initial nutrient levels of (*a*) $n_0 = 100\ \mu gN\ L^{-1}$ and $p_0 = 10\ \mu gP\ L^{-1}$, and (*b*) $n_0 = 72\ \mu gN\ L^{-1}$ and $p_0 = 36\ \mu gP\ L^{-1}$.

Solution: Mass balances for algae, phosphorus, and nitrogen can be written as

$$\frac{da}{dt} = k_g a \qquad \frac{dp}{dt} = -a_{pa} k_g a \qquad \frac{dn}{dt} = -a_{na} k_g a$$

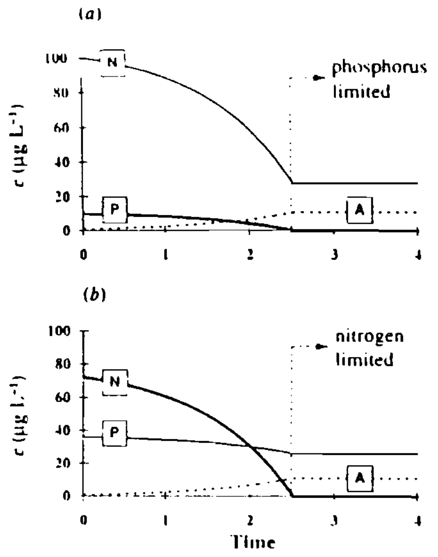

FIGURE 28.6
Algal growth in a batch reactor starting with initial N:P ratios that lead to (*a*) phosphorus limitation (initial N:P = 10) and (*b*) nitrogen limitation (initial N:P = 2).

which can be solved for

$$a = a_0 e^{k_g t} \qquad p = p_0 - a_{pa} a_0 (e^{k_g t} - 1) \qquad n = n_0 - a_{na} a_0 (e^{k_g t} - 1)$$

(*a*) The solutions for the first case are depicted in Fig. 28.6*a*. Because it is in relatively short supply (N:P = 10 > 7.2), the phosphorus runs out first at approximately t = 2.5 d. At this point the algae can no longer grow and excess nitrogen remains in the water.
(*b*) The second case, where nitrogen is in short supply (N:P = 2 < 7.2), is depicted in Fig. 28.6*b*. For this case the nitrogen runs out first. Again, at this point, the algae can no longer grow. However, now excess phosphorus remains in the water.

As displayed in Table 28.3, sewage is generally enriched in phosphorus. Therefore water bodies dominated by wastewater effluents tend to be nitrogen limited. Similarly estuaries tend to be deficient in nitrogen and hence are usually nitrogen limited. In contrast those systems subject to phosphorus removal and nonpoint-source input are generally phosphorus limited.

TABLE 28.3
N:P ratios for point, nonpoint, and marine waters (data from Thomann and Mueller 1987, Omernik 1977, and Goldman et al. 1973)

Source type	TN/TP[1]	IN/IP[1]	Limiting nutrient
Raw sewage	4	3.6	Nitrogen
Activated sludge	3.4	4.4	Nitrogen
Activated sludge plus nitrification	3.7	4.4	Nitrogen
Activated sludge plus phosphorus removal	27.0	22.0	Phosphorus
Activated sludge plus nitrogen removal	0.4	0.4	Nitrogen
Activated sludge plus nitrogen and phosphorus removal	3.0	2.0	Nitrogen
Nonpoint sources	28	25	Phosphorus
Marine waters	—	2	Nitrogen

[1] TN/TP = total nitrogen/total phosphorus; [1] IN/IP = inorganic nitrogen/inorganic phorphorus.

Although Table 28.3 provides general patterns, individual natural waters must be assessed on a case-by-case basis. Consequently mathematical models have been developed to simulate the eutrophication process.

PROBLEMS

28.1. A sewage treatment pond has the following characteristics:

Residence time = 3 wk
Area = 1×10^5 m^2
Mean depth = 2 m

The inflow concentrations of nitrogen and phosphorus are 50 and 5 mg L^{-1}, respectively. These inputs result in a level of 200 mgChl*a* m^{-3} of phytoplankton in the pond. The phytoplankton are removed by settling at a rate of 0.5 m d^{-1}. Also, the nitrogen is removed by denitrification at a rate of 0.05 d^{-1}. Compute the following:
(*a*) The steady-state nitrogen and phosphorus concentrations in the pond
(*b*) Based on the nitrogen-to-phosphorus ratio, which is the limiting nutrient?

Note that the chlorophyll-to-phosphorus ratio in the phytoplankton is 1.

28.2. A stretch of a river (U = 1 m s^{-1}) has a uniform, net photosynthesis rate of 10 gO m^{-2} d^{-1}. The boundary conditions of inorganic nitrogen and phosphorus at the head end of the stretch are 20 mgN L^{-1} and 4 mgP L^{-1}, respectively. If plant activity is the only significant source or sink of nutrients, determine and plot the N:P ratio for the stretch until one of the nutrients runs out.

28.3. Right after thermal stratification is established, a lake has available phosphorus and available nitrogen (mostly nitrate) concentrations of 10 μgP L^{-1} and 100 μgN L^{-1}. If the chlorophyll-to-carbon ratio is 25 μgChl mg^{-1}: (*a*) How much biomass could potentially be produced by photosynthesis in μgChl L^{-1}? (*b*) How much carbon in mgC L^{-1}? (*c*) How much oxygen would be required in mgO L^{-1} to nitrify the organic nitrogen produced by photosynthesis?

28.4. A town of 20,000 people discharges raw sewage into a river with a flow of 1 cms. If the river has 10 μgP L^{-1} and 100 μgN L^{-1} prior to the addition of the sewage, what is the N:P ratio at the mixing point? What is the limiting nutrient above and below the discharge?